CSE 105, Winter 2022 - Homework 2

Due: Monday (1/24/2022) 11:59 PM

Instructions

Upload a single file to Gradescope for each group. All group members’ names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Sipser Sections 1.1 - 1.3

Key Concepts Deterministic finite automata (DFA), state diagram, computation trace, accept/reject, the language of an automaton, regular language, union of languages, concatenation of languages, star of a language, closure of the class of regular languages under certain operations, nondeterministic finite automata (NFA), Regular Expressions.
**Problem 1 (10 points)**

You have an incoming bitstream (sequence of 0s and 1s) that might be truncated at any time. Your goal is to construct a DFA which identifies if the current input, at any given time, belongs to a certain language \( L \) or not.

That is, if \( x \) is the current bitstream, then the DFA should be in an accepting state if \( x \) is in \( L \), and in a rejecting state if \( x \) is not in \( L \). After another bit of the input is read, the DFA should update to an appropriate state, and so on.

The specific language we are interested in is the following. Consider the bitstream \( x \) as a number in binary, reading bits from left to right. So for example, 0111 is 7, and 1000 is 8. The language is:

\[
L = \{ x \mid x \text{ is divisible by 4 when considered as a number in binary} \}
\]

(a) Draw the state diagram of the DFA. Justify why your construction is correct.

(b) Write the formal definition of the DFA \( M = (Q, \Sigma, \delta, q_0, F) \). Use a table to define \( \delta \).

(If you need, you can use http://madebyevan.com/fsm/ to draw your diagram)

**Problem 2 (10 points)**

Recall the following operations on languages (sets of strings) \( A, B \):

- Union \( A \cup B = \{ x \mid x \in A \text{ or } x \in B \} \)
- Concatenation \( A \circ B = \{ xy \mid x \in A \text{ and } y \in B \} \)
- Star \( A^* = \{ x_1 x_2 \ldots x_k \mid k \in \mathbb{Z} \text{ and } k \geq 0 \text{ and each } x_i \in A \} \)

For each of the following languages over the alphabet \{a, b\}, answer the following questions:

(i) Is \( \varepsilon \) (the empty string) in the set?

(ii) What is an example of a string of length at least 2 that is in the language, or why isn't there such an example?

(iii) What is an example of a string of length at least 2 that is not in the language, or why isn't there such an example?

1. \( \{a\} \circ \{b\}^* \)
2. \( \{ab, ba\}^* \)
3. \( \{a, b\} \cup \{aa, bb\} \)
**Problem 3 (10 points)**

Prove that every Non-deterministic Finite Automaton (NFA) has an equivalent Deterministic Finite Automaton (DFA), accepting the same language. We will go over the proof at a high level in class, and also the book contains the proof. Your goal is to understand this proof and then write, using your own words, a detailed proof.

**Problem 4 (10 points)**

Convert the following two NFAs to equivalent DFAs.

(a) ![Diagram](image1)

(b) ![Diagram](image2)

**Problem 5 (10 points)**

Let \( L \) be a regular language over the alphabet \( \{0,1\} \). Define the language \( \text{Prefix}_L \) to be all the prefixes of words in \( L \). That is:

\[
\text{Prefix}_L = \{ x \in \{0,1\}^* : \exists y \in \{0,1\}^* \text{ such that } x \cdot y \in L \}
\]

Prove that \( \text{Prefix}_L \) is also regular.