Three-View Geometry
Trifocal Plane
Trifocal Tensor

- 3x3x3 tensor
- 27 elements, 18 degrees of freedom
  - 33 degrees of freedom (3 camera projection matrices) minus 15 degrees of freedom (3D projective transformation)
- Uses tensor notation
  - Einstein summation
- Retrieve fundamental matrices and camera projection matrices
## Trilinear Relations

<table>
<thead>
<tr>
<th>Correspondence</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>three points</td>
<td>$x^i x'^j x''^k \epsilon_{jqs} \epsilon_{krt} T_{i}^{qr} = 0_{st}$</td>
</tr>
<tr>
<td>two points, one line</td>
<td>$x^i x'^j l''<em>r \epsilon</em>{jqs} T_{i}^{qr} = 0_s$</td>
</tr>
<tr>
<td>one point, two lines</td>
<td>$x^i l''_q l''<em>r T</em>{i}^{qr} = 0$</td>
</tr>
<tr>
<td>three lines</td>
<td>$l_p l''<em>q l''<em>r \epsilon</em>{piw} T</em>{i}^{qr} = 0^w$</td>
</tr>
</tbody>
</table>
3 Points

• Point-Point-Point
2 Points, 1 Line

- **Point-Line-Point**
  - Note: image line must pass through corresponding image point
1 Point, 2 Lines

• Point-Line-Line
  – Note: image lines do not need to correspond, but must pass through corresponding image points
3 Lines

- Line-Line-Line
Trifocal Tensor, Minimal Solution

• 6 image point correspondences between three images
  – The last 4 image points must be in general position
    • 6 choose 4 = 15 combinations (rearrange, if needed)
• Carlsson-Weinshall duality (interchange camera and points)
  – Dualize first 2 image points
    • Results in 3 dual point correspondences
    • Additional 4 dual point correspondences are 2D projective basis
  – Calculate reduced dual fundamental matrix (5 degrees of freedom) from 7 dual point correspondences
    • 1 or 3 solutions
• Solve for three camera projection matrices
  – 5 3D points are 3D projective basis
  – 1 or 3 solutions for 6th 3D point using dual fundamental matrix
• Calculate trifocal tensor from three camera projection matrices
  – 1 or 3 solutions
Trifocal Tensor Estimation

- **Outlier rejection (MSAC)**
  - 6-point algorithm (previous slide)
  - Sampson error

- **Linear estimate (DLT)**
  \[ x^q l^l qr T^r_i = 0 \]
  - Enforce constraints on resulting trifocal tensor
    - 18 degrees of freedom

- **Nonlinear estimate (L-M)**
  - Cameras
    - Retrieve camera projection matrices from trifocal tensor
      - First camera projection matrix is canonical camera
      - Parameterize second and third cameras
        » Minimal parameterization
          - 18 parameters (18 degrees of freedom)
        » Non-minimal parameterization
          - Second and third camera projection matrices using parameterization of homogeneous vectors

  - 3D scene points
    - Triangulation of Sampson correct points

  - Adjust second and third camera parameters, and 3D scene points such that reprojection error is minimized
Mapping Under Trifocal Tensor

- Example: lines in any two images to line in the other image

\[ l_p l'_q l''_r e^{piw} T_i^{qr} = 0^w \]
Mapping Under Trifocal Tensor

• Example: points in any two images to point in the other image

\[ x'^i x'^j x'^'k \epsilon_{js} \epsilon_{krt} T_i^{qr} = 0_{st} \]