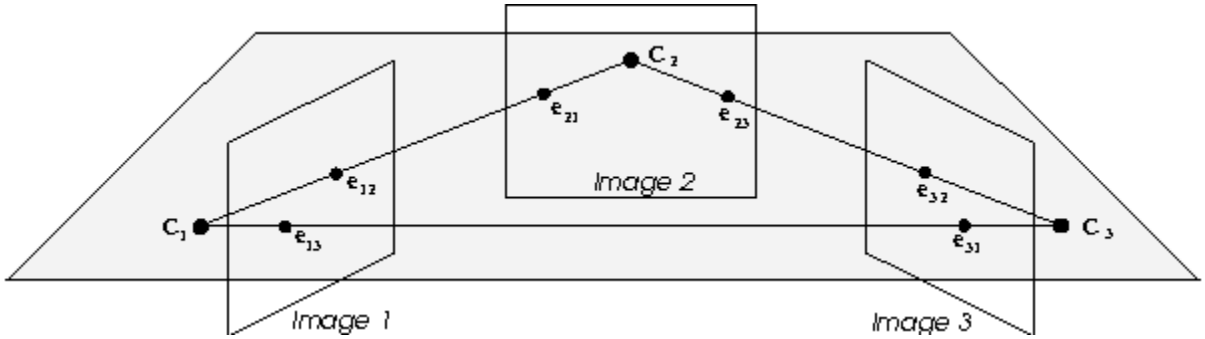
Three-View Geometry

Computer Vision II

CSE 252B

Trifocal Plane



Trifocal Tensor

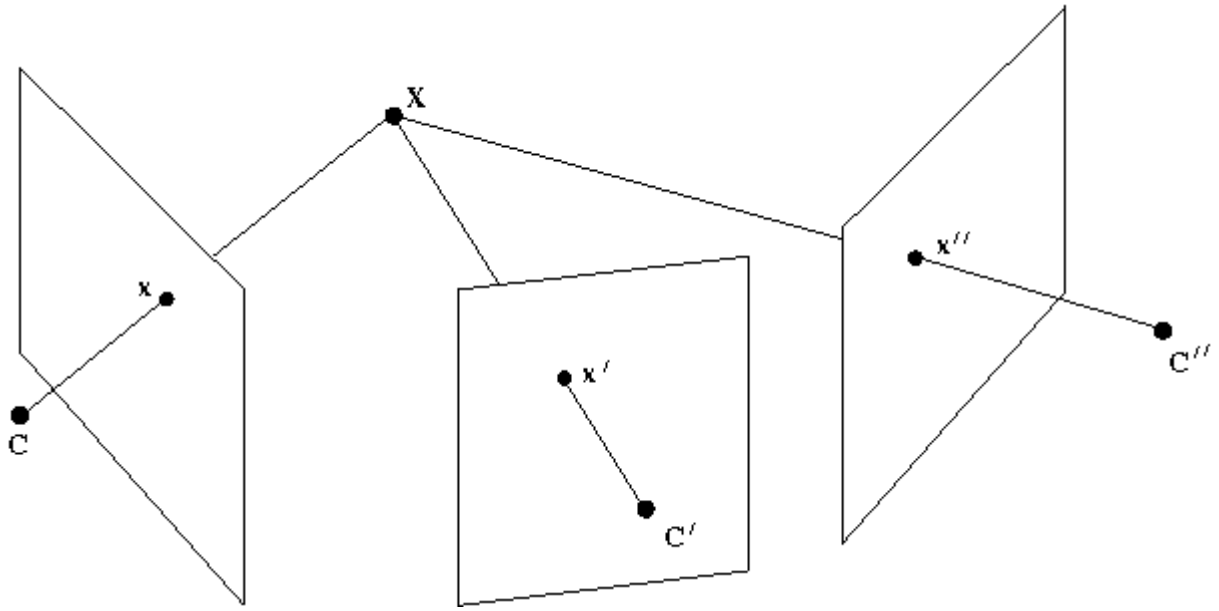
- 3x3x3 tensor
- 27 elements, 18 degrees of freedom
  - 33 degrees of freedom (3 camera projection matrices) minus 15 degrees of freedom (3D projective transformation)
- Uses tensor notation
  - Einstein summation
- Retrieve fundamental matrices and camera projection matrices

Trilinear Relations

Correspondence	Relation
three points	$x^i x'^j x''^k \epsilon_{jqs} \epsilon_{krt} \mathcal{T}_i^{qr} = 0_{st}$
two points, one line	$x^i x'^j l''_r \epsilon_{jqs} \mathcal{T}_i^{qr} = 0_s$
one point, two lines	$x^i l'_q l''_r \mathcal{T}_i^{qr} = 0$
three lines	$l_p l'_q l''_r \epsilon^{piw} \mathcal{T}_i^{qr} = 0^w$

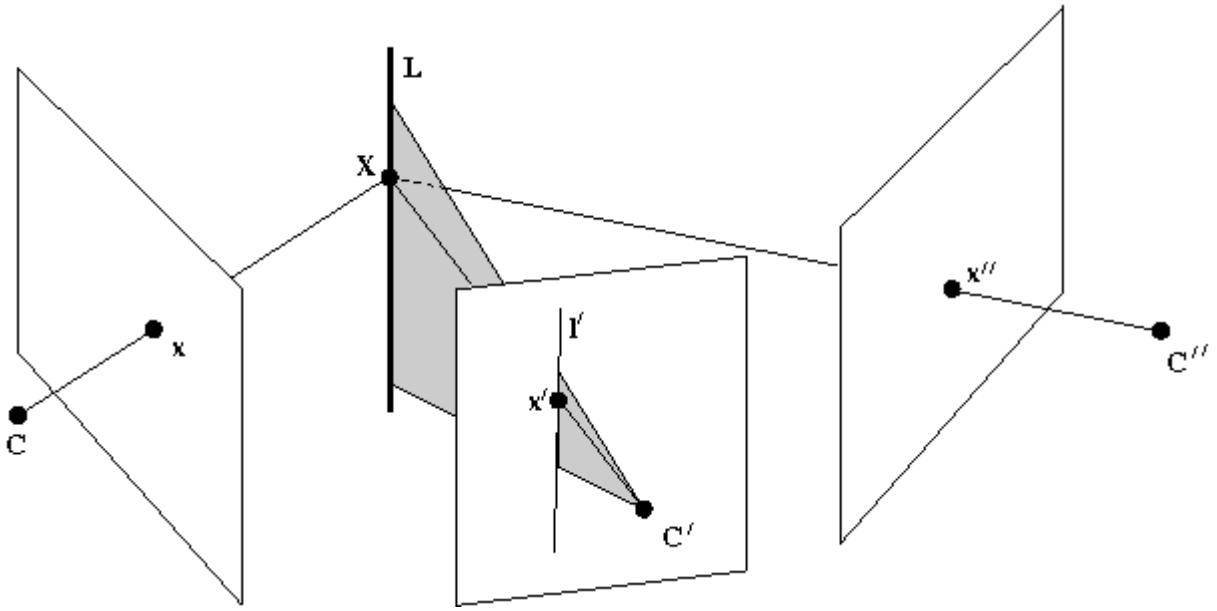
3 Points

- Point-Point-Point



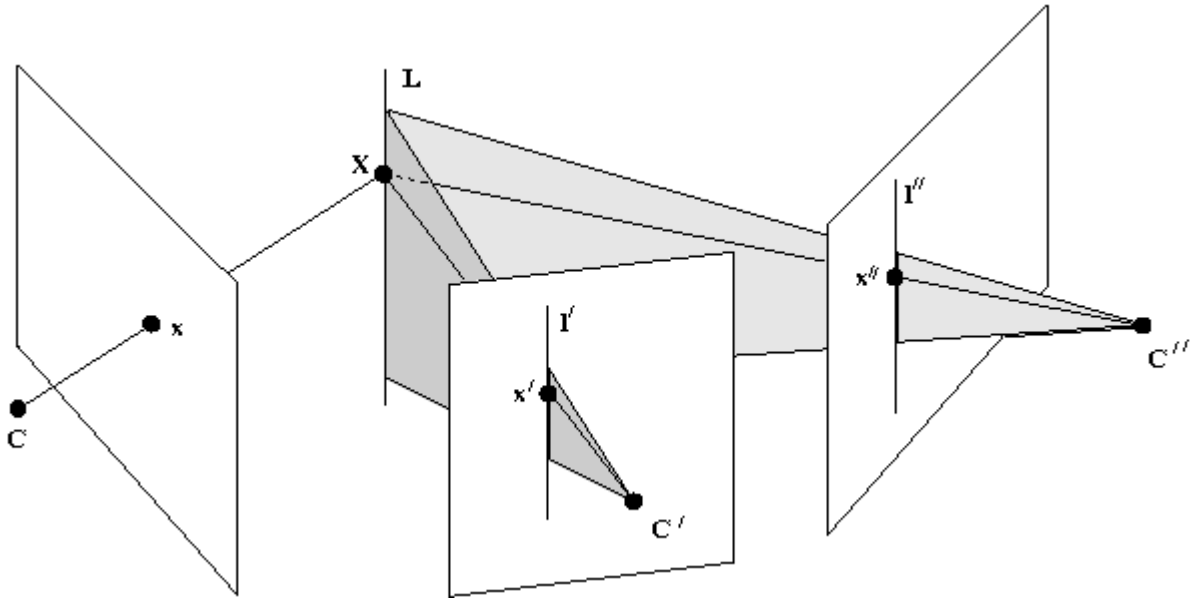
2 Points, 1 Line

- Point-Line-Point
  - Note: image line must pass through corresponding image point



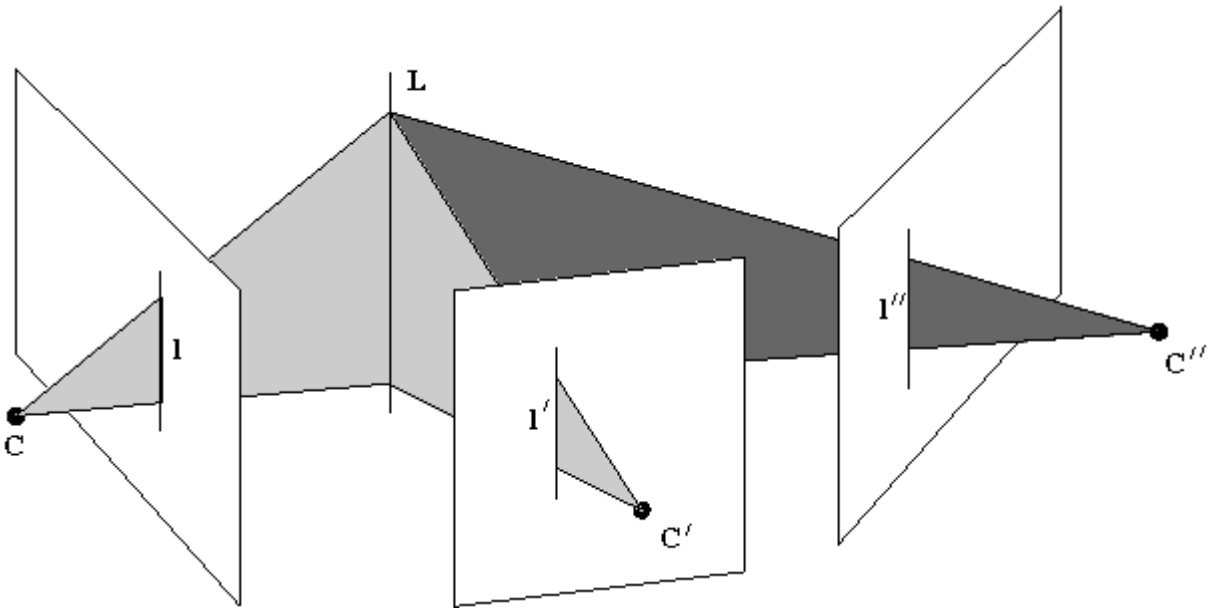
1 Point, 2 Lines

- Point-Line-Line
  - Note: image lines do not need to correspond, but must pass through corresponding image points



3 Lines

- Line-Line-Line



Trifocal Tensor, Minimal Solution

- 6 image point correspondences between three images
  - The last 4 image points must be in general position
    - 6 choose 4 = 15 combinations (rearrange, if needed)
- Carlsson-Weinshall duality (interchange camera and points)
  - Dualize first 2 image points
    - Results in 3 dual point correspondences
    - Additional 4 dual point correspondences are 2D projective basis
  - Calculate reduced dual fundamental matrix (5 degrees of freedom) from 7 dual point correspondences
    - 1 or 3 solutions
- Solve for three camera projection matrices
  - 5 3D points are 3D projective basis
  - 1 or 3 solutions for 6th 3D point using dual fundamental matrix
- Calculate trifocal tensor from three camera projection matrices
  - 1 or 3 solutions

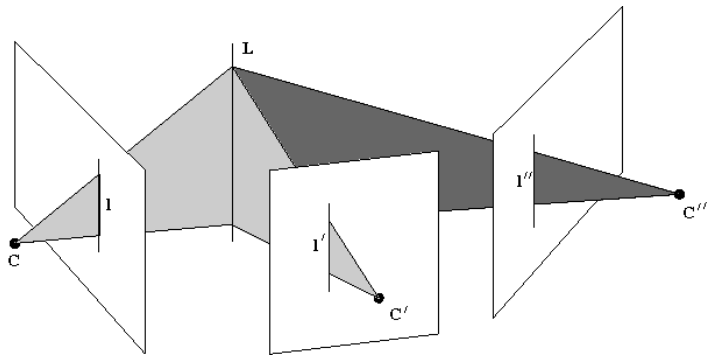
Trifocal Tensor Estimation

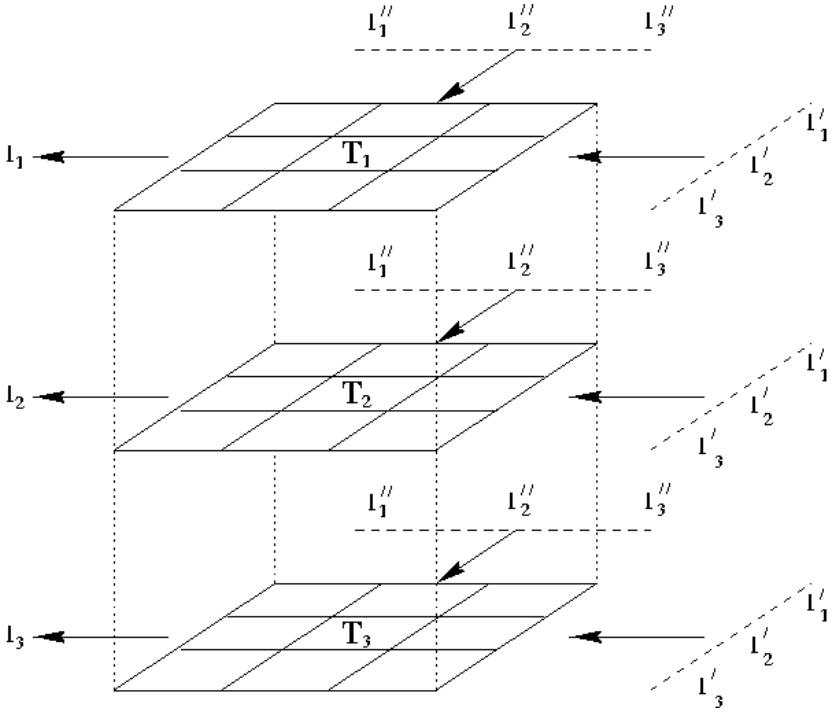
- Outlier rejection (MSAC)
  - 6-point algorithm (previous slide)
  - Sampson error
- Linear estimate (DLT) $x^i l'_q l''_r \mathcal{T}_i^{qr} = 0$
  - Enforce constraints on resulting trifocal tensor
    - 18 degrees of freedom
- Nonlinear estimate (L-M)
  - Cameras
    - Retrieve camera projection matrices from trifocal tensor
      - First camera projection matrix is canonical camera
      - Parameterize second and third cameras
        - » Minimal parameterization
          - 18 parameters (18 degrees of freedom)
        - » Non-minimal parameterization
          - Second and third camera projection matrices using parameterization of homogeneous vectors
  - 3D scene points
    - Triangulation of Sampson correct points
  - Adjust second and third camera parameters, and 3D scene points such that reprojection error is minimized

Mapping Under Trifocal Tensor

- Example: lines in any two images to line in the other image

$$l_p l'_q l''_r \epsilon^{piw} \mathcal{T}_i^{qr} = 0^w$$





Mapping Under Trifocal Tensor

- Example: points in any two images to point in the other image

$$x^i x'^j x''^k \epsilon_{jqs} \epsilon_{krt} \mathcal{T}_i^{qr} = 0_{st}$$