

- (1) This is an open book, open notes exam. You are free to consult any text book or notes. **You are not allowed to consult with any other person.**
- (2) If you need any clarification, please post a private message to the instructors on Piazza.
- (3) Remember that your work is graded on the *clarity* of your writing and explanation as well as the validity of what you write.
- (4) This is a one-hour exam.

- (1) (10 points) Draw the decision boundary for the 1-nearest neighbor classifier on the following labeled data points in 2 dimensions.

$$((1, 2), 1), ((-3, 4), 2), ((-2, -1), 3)$$

For full credit, write down the equations for each segment of the decision boundary. Clearly label all the decision regions with the label assigned by the 1-nearest neighbor classifier.

- (2) Suppose we have instances X drawn from an instance space \mathcal{X} and labels Y drawn from $\{0, 1\}$. In class and the homework, we talked about how the data distribution D is a joint distribution over $X \times Y$ and can be factorized into the marginal over X times the conditional distribution of Y given X as: $D(X = x, Y = y) = \mu(X = x) \cdot \eta(Y = y|X = x)$. It turns out that D can also be factorized in a different manner:

$$D(X = x, Y = y) = \pi(Y = y) \cdot f(X = x|Y = y)$$

where $\pi(Y = y)$ is the marginal over Y and $f(X = x|Y = y)$ is the conditional likelihood of X given a specific value of Y .

- (a) (5 points) Write down expressions for $\pi(Y = y)$ and $f(X = x|Y = y)$ as functions of $\mu(X = x)$ and $\eta(Y = y|X = x)$.

- (b) (5 points) Suppose now that we change $f(X = x|Y = y)$ while keeping $\pi(Y = y)$ fixed. Does the Bayes Optimal classifier for D change? If yes, give an example where it happens. If no, give a brief justification.