CSE 250B Winter 2020 - Quiz 4

Mar 6, 2020 - 30 Minutes **Name:** **PID:**

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(1) This is a closed book, closed notes exam. Switch off your cell phone and do not communicate with anyone other than an exam proctor.
(2) Start writing when instructed. Stop writing when your time is up.
(3) Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) In each of the following cases, state if the statement is true or false. In either case, justify your answer.
  (a) (5 points) Let X be any arbitrary discrete random variable, and let $Z = X^2$. Then, $H(X) = H(Z)$.

False
Consider discrete variable X, with $P(X = -1) = P(X = 1) = 0.5$.
For variable X^2, $P(X^2 = 1) = 1$

$$H(X) = -\sum_{x=\{1,-1\}} P(X = x) \log P(X = x) = 1$$

$$H(Z) = H(X^2) = -P(X^2 = 1) \log P(X^2 = 1) = 0$$

  (b) (5 Points) If $K(x, z)$ and $L(x, z)$ are kernels, then the function:

$$M(x, z) = \frac{K(x, z)}{L(x, x)L(z, z)}$$

is also a kernel.

True
Let $\psi(x)$ be the feature map for kernel K, i.e. $K(x, z) = \langle \psi(x), \psi(z) \rangle$.
Then we have

$$\begin{aligned} M(x, z) &= \frac{K(x, z)}{L(x, x)L(z, z)} \\ &= \frac{\langle \psi(x), \psi(z) \rangle}{L(x, x)L(z, z)} \\ &= \langle \omega(x), \omega(z) \rangle \end{aligned}$$

where $\omega(x) = \frac{\psi(x)}{L(x,x)}$. Therefore, M is a kernel with $\omega(x)$ as the feature map.

(c) (5 points)Recall that two classifiers C and C' are equal if for all x in $\mathbb{R}^d$, $C(x) = C'(x)$. If two decision trees have zero true error on a data distribution, then they are equal.

False Consider a distribution where the input X is a 1-d binary random variable with probability $P(X = 1) = P(X = 0) = 1/2$, and the label $Y = X$.
The following two decision trees

$$D_1 = \begin{cases} 0 & X < 0.5 \\ 1 & otherwise \end{cases}$$

$$D_2 = \begin{cases} 0 & X < 0.25 \\ 1 & otherwise \end{cases}$$

both have zero true error on the distribution. However, D_1 and D_2 are not equal, e.g. $D_1(0.4) = 0 \neq 1 = D_2(0.4)$.