Photometric Stereo

Introduction to Computer Vision I
CSE 152A
Lecture 4
Announcements

• Assignment 0 is due today, 11:59 PM
• Assignment 1 will be released today
  – Due Jan 27, 11:59 PM
• Quiz 1 is Jan 29
• Reading:
  – Szeliski
    • Section 13.1.1
Shading reveals 3D surface geometry
Two shape-from-X methods that use shading

• Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

• Photometric stereo: Single viewpoint, multiple images under different lighting.

BRDF (four dimensional function)
Photometric Stereo Rigs: One viewpoint, changing lighting
An example of photometric stereo

- Surface
  - (albedo texture map)
- Surface normals
- Albedo

Example images showing the effect of different lighting conditions on a human face.
Photometric stereo

- Single viewpoint, multiple images under different lighting
  1. General BRDF, known lighting
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting
1. Photometric Stereo: General BRDF and Reflectance Map
BRDF

- Bi-directional Reflectance Distribution Function
  \[ \rho(\theta_{\text{in}}, \phi_{\text{in}} ; \theta_{\text{out}}, \phi_{\text{out}}) \]

- Function of
  - Incoming light direction:
    \[ \theta_{\text{in}} , \phi_{\text{in}} \]
  - Outgoing light direction:
    \[ \theta_{\text{out}} , \phi_{\text{out}} \]

- Ratio of incident irradiance to emitted radiance
Coordinate system

Surface: \( s(x,y) = (x,y, f(x,y)) \)

Tangent vectors:
\[
\frac{\partial s(x,y)}{\partial x} = \left(1, 0, \frac{\partial f}{\partial x}\right)
\]
\[
\frac{\partial s(x,y)}{\partial y} = \left(0, 1, \frac{\partial f}{\partial y}\right)
\]

Normal vector
\[
n = \frac{\partial s}{\partial x} \times \frac{\partial s}{\partial y} = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1\right)
\]
Gradient Space $(p,q)$

Gradient Space: $(p,q)$

$p = \frac{\partial f}{\partial x}, \quad q = \frac{\partial f}{\partial y}$

Normal vector

$$\mathbf{n} = \frac{\partial s}{\partial x} \times \frac{\partial s}{\partial y} = \begin{pmatrix} -\frac{\partial f}{\partial x}, & -\frac{\partial f}{\partial y}, & 1 \end{pmatrix}^T$$

$$\mathbf{\hat{n}} = \frac{1}{\sqrt{p^2 + q^2 + 1}} \begin{pmatrix} -p, & -q, & 1 \end{pmatrix}^T$$
Image Formation

For a given point A on the surface \( a \), the image irradiance \( E(x,y) \) is a function of

1. The BRDF at A
2. The surface normal at A
3. The direction of the light source
Reflectance Map

Let the BRDF be the same at all points on the surface, and let the light direction $s$ be a constant.

1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have $E(p,q)$.
Example Reflectance Map: Lambertian surface

\[ E(p,q) \]

For lighting from front
LAMBERTIAN REFLECTANCE MAP

\[ E = L \rho \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2} \sqrt{1 + p_s^2 + q_s^2}} \]

\( p_s = -2 \quad q_s = -1 \)

Light Source Direction, expressed in gradient space.
What does the intensity (irradiance) of one pixel in one image tell us?

It constrains the surface normal projecting to that point to a curve.
Two Light Sources  
Two reflectance maps

A third image would disambiguate match

E.g., Normal lies on this curve

i=0.5

i=0.4
Three Source Photometric stereo:  
Step 1

Offline:
Using source directions & BRDF, construct reflectance map for each light source direction. $R_1(p,q)$, $R_2(p,q)$, $R_3(p,q)$

Online:
1. Acquire three images with known light source directions. $E_1(x,y)$, $E_2(x,y)$, $E_3(x,y)$
2. For each pixel location $(x,y)$, find $(p,q)$ as the intersection of the three curves
   $R_1(p,q) = E_1(x,y)$
   $R_2(p,q) = E_2(x,y)$
   $R_3(p,q) = E_3(x,y)$
3. This is the surface normal at pixel $(x,y)$. Over image, the normal field is estimated
Normal Field
Plastic Baby Doll: Normal Field
Next step:
Go from normal field to surface
Recovering the surface $f(x,y)$

Many methods: Simplest approach

1. From estimate $\mathbf{n} = (n_x, n_y, n_z)$, $p = n_x/n_z$, $q = n_y/n_z$
2. Integrate $p = df/dx$ along a row $(x,0)$ to get $f(x,0)$
3. Then integrate $q = df/dy$ along each column starting with value of the first row
What might go wrong?

• Height $z(x,y)$ is obtained by integration along a curve from $(x_0, y_0)$.

$$z(x, y) = z(x_0, y_0) + \int_{(x_0, y_0)}^{(x, y)} (pdx + qdy)$$

• If one integrates the derivative field along any closed curve, one expects to get back to the starting value.

• Might not happen because of noisy estimates of $(p, q)$
What might go wrong?

Integrability. If \( f(x,y) \) is the height function, we expect that

\[
\frac{\partial}{\partial y} \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}
\]

In terms of estimated gradient space \((p,q)\), this means:

\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]

But since \( p \) and \( q \) were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold
Horn’s Method
[“Robot Vision, B.K.P. Horn, 1986”]

- Formulate estimation of surface height $z(x,y)$ from gradient field by minimizing cost functional:

$$\iint (z_x - p)^2 + (z_y - q)^2 \, dx \, dy$$

where $(p,q)$ are estimated components of the gradient while $z_x$ and $z_y$ are partial derivatives of best fit surface.

- Solved using calculus of variations – iterative updating

- $z(x,y)$ can be discrete or represented in terms of basis functions

- Integrability is naturally satisfied
2. Photometric Stereo: Lambertian Surface, Known Lighting
At image location \((u,v)\), the intensity of a pixel \(x(u,v)\) is:

\[
e(u,v) = [a(u,v) \hat{n}(u,v)] \cdot [s_0 \hat{s}]
\]

\[
= b(u,v) \cdot s
\]

where

- \(a(u,v)\) is the albedo of the surface projecting to \((u,v)\)
- \(\hat{n}(u,v)\) is the direction of the surface normal
- \(s_0\) is the light source intensity
- \(\hat{s}\) is the direction to the light source
Lambertian Photometric stereo

• If the light sources $s_1$, $s_2$, and $s_3$ are known, then we can recover $b$ from as few as three images. (Photometric Stereo: Silver 80, Woodham 81).

\[
\begin{bmatrix}
  e_1 \\
  e_2 \\
  e_3
\end{bmatrix} = b^T \begin{bmatrix}
  s_1 \\
  s_2 \\
  s_3
\end{bmatrix}
\]

• i.e., we measure $e_1$, $e_2$, and $e_3$ and we know $s_1$, $s_2$, and $s_3$. We can then solve for $b$ by solving a linear system.

\[
b^T = \begin{bmatrix}
  e_1 & e_2 & e_3
\end{bmatrix} \begin{bmatrix}
  s_1 \\
  s_2 \\
  s_3
\end{bmatrix}^{-1}
\]

• Normal $\hat{n} = b/|b|$ and albedo $a = |b|$
What if we have more than 3 Images?

Linear Least Squares

\[
\begin{bmatrix}
e_1 & e_2 & e_3 & \ldots & e_n
\end{bmatrix} = b^T [s_1 \ s_2 \ s_3 & \ldots & s_n ]
\]

Rewrite as

\[
e = Sb
\]

where

- \( e \) is \( n \) by \( 1 \)
- \( b \) is \( 3 \) by \( 1 \)
- \( S \) is \( n \) by \( 3 \)

Solving for \( b \) gives

\[
b = (S^T S)^{-1} S^T e
\]

Let the residual be

\[
r = e - Sb
\]

Squaring this:

\[
r^2 = r^T r = (e - Sb)^T (e - Sb) = e^T e - 2b^T S^T e + b^T S^T S b
\]

\[(r^2)_b = 0 \quad \text{zero derivative is a necessary condition for a minimum, or}
\]

\[-2S^T e + 2S^T S b = 0;\]
Input Images
Recovered albedo
Recovered normal field
Surface recovered by integration
An example of photometric stereo

Images with known associated light sources

Albedo

Surface (from normals)

Surface (albedo texture map)
Next Lecture

• Image filtering

• Reading:
  – Szeliski
    • Sections 3.2 and 3.3.1