Photometric Image Formation

Introduction to Computer Vision I
CSE 152A
Lecture 3
Announcements

• Assignment 0 is due Jan 13, 11:59 PM
• Assignment 1 will be released Jan 13
  – Due Jan 27, 11:59 PM
• Reading:
  – Szeliski
    • Section 2.2
Geometric image formation
Photometric image formation
Beyond the pinhole Camera
Getting more light – Bigger Aperture
Pinhole Camera Images with Variable Aperture

- 2 mm
- 1 mm
- 0.6 mm
- 0.35 mm
- 0.15 mm
- 0.07 mm
The reason for lenses
We need light, but big pinholes cause blur.
• Rotationally symmetric about optical axis
• Spherical interfaces
Thin Lens: Center

• All rays that enter lens along line pointing at \( O \) emerge in same direction
Thin Lens: Focus

Parallel lines pass through the focus F
Thin Lens: Image of Point

All rays passing through lens and starting at P converge upon P'

So light gather capability of lens is given the area of the lens and all the rays focus on P' instead of become blurred like a pinhole
Thin Lens: Image of Point

Relation between depth of Point (-Z) and the depth where it focuses (Z')

\[
\frac{1}{Z'} - \frac{1}{Z} = \frac{1}{f}
\]
Thin Lens: Image Plane

A price: Whereas the image of $P$ is in focus, the image of $Q$ is not.
Thin Lens: Aperture

- Smaller Aperture -> Less Blur
- Pinhole -> No Blur
Photometric image formation

- Light incident on a given pixel
Measuring Angle

- The **solid angle** subtended by an object from a point $P$ is the area of the projection of the object onto the unit sphere centered at $P$.
- Definition is analogous to projected angle in 2D.
- Measured in **steradians**, $sr$.
- If I am at $P$ and I look out, the solid angle tells me how much of my view is filled with an object.
Radiance

- Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle
  - Units: watts per square meter per steradian, W/m²/sr = W m⁻² sr⁻¹
  \[
  L(X, \theta, \phi) = \frac{P}{(dA \cos \theta)d\omega}
  \]

Irradiance

- Total power arriving at the surface (from all incoming angles)
  - Units: power per unit area, W/m² = W m⁻²
  \[
  E(X) = \int_{\text{hemisphere}} L(X, \theta, \phi) \cos \theta d\omega
  \]
Visible Light Spectrum
Camera sensor

- Measured pixel intensity is a function of irradiance $E$ integrated over
  - Pixel’s area $(x, y)$
  - range of wavelengths $\lambda$
  - some period of time $t$

$$I = \int \int \int \int E(x, y, \lambda, t)s(x, y)q(\lambda) \, dx \, dy \, d\lambda \, dt$$

- Ideally, the camera response function $R$ is linear to the radiance, but it may not be

$$I = R\left(\int \int \int \int E(x, y, \lambda, t)s(x, y)q(\lambda) \, dx \, dy \, d\lambda \, dt\right)$$
Image irradiance is proportional scene radiance

For a camera with a thin lens, it can be shown that

$$E(x) = k_L L$$

where

- $E(x)$ is the image irradiance at point $x$
- $L$ is the radiance coming from a scene point projecting to image point $x$
- $k_L$ is a proportionality constant that may depend on the lens is may be a function of $x$

Combined with linear sensor model, we have

$$I = k_c k_L L$$

In other words, the measured pixel intensity is proportional to the radiance
Color Cameras

Eye:

Three types of Cones

Cameras:

1. Filter wheel
2. Prism (with 3 sensors)
3. Filter mosaic

... and X3
Filter wheel

Rotate multiple filters in front of lens
Allows more than 3 color bands

Only suitable for static scenes
Prism color camera

Separate light in 3 beams using dichroic prism
Requires 3 sensors & precise alignment
Good color separation
Filter mosaic

Coat filter directly on sensor

Demosaicimg (obtain full colour & full resolution image)
Color CMOS sensor
Foveon’s X3

better image quality

smarter pixels

VPS Enables a Foveon X3 image sensor to be addressed in variable resolutions.
Light

• Special light sources
  – Point sources
  – Distant point sources
  – Strip sources
  – Area sources

• Note, if light is very far away, then view light as coming from a direction in 3D
  – Directions in 3D can be represented as a point on a sphere
  – Distant lighting can be viewed as a function giving brightness over a sphere
Light at surfaces

Many effects when light strikes a surface -- could be:

- Reflected
  - Mirror
- Transmitted
  - Skin, glass
- Scattered
  - Milk
- Travel along the surface and leave at some other point
- Absorbed

We will assume:

- All the light leaving a point is due to that arriving at that point
- Surfaces don’t fluoresce
  - e.g. scorpions, detergents
- Surfaces don’t emit light (i.e. are cool)
Light at surfaces
BRDF

• Bi-directional Reflectance Distribution Function
  \[ \rho(\theta_{\text{in}}, \phi_{\text{in}} ; \theta_{\text{out}}, \phi_{\text{out}}) \]

• Function of
  – Incoming light direction:
    \[ \theta_{\text{in}}, \phi_{\text{in}} \]
  – Outgoing light direction:
    \[ \theta_{\text{out}}, \phi_{\text{out}} \]

• Ratio of incident irradiance to emitted radiance
Specular reflection

- Ideal specular reflection is mirror reflection
  - Perfectly smooth surface
  - Incoming light ray is bounced in single direction
  - Angle of incidence equals angle of reflection
Specular Reflection: Smooth Surface

- \( \mathbf{N}, \omega_i, \omega_o \) are coplanar
- \( \theta_i = \theta_o \)

\[
\omega_o = 2(\omega_i \cdot \mathbf{N}) \mathbf{N} - \omega_i
\]

Speculum – Latin for “Mirror”
Diffuse surface

- Ideal diffuse material reflects light equally in all directions
- View-independent
- Matte, not shiny materials
  - Paper
  - Unfinished wood
  - Unpolished stone
Diffuse reflection

• Beam of parallel rays shining on a surface
  – Area covered by beam varies with the angle between the beam and the normal
  – The larger the area, the less incident light per area
  – Incident light per unit area is proportional to the cosine of the angle between the normal and the light rays
• Object darkens as normal turns away from light
• Lambert’s cosine law (Johann Heinrich Lambert, 1760)
• Diffuse surfaces are also called Lambertian surfaces
Lambertian (Diffuse) Reflection

The intensity (irradiance) $I(u,v)$ of a pixel at $(u,v)$ is:

$$I(u,v) = a(u,v)\hat{n}(u,v) \cdot s_0 \hat{s}$$

- $a(u,v)$ is the albedo of the surface projecting to $(u,v)$
- $\hat{n}(u,v)$ is the direction of the surface normal
- $s_0$ is the light source intensity
- $\hat{s}$ is the direction to the light source

Do not allow angles less than 0 (light is behind surface)

$$\cos(\theta) = n^T L$$
$$\cos^+(\theta) = \max(0, n^T L)$$
Glossy surface

- Assume surface composed of small mirrors with random orientation (micro-facets)
- Smooth surfaces
  - Micro-facet normals close to surface normal
  - Sharp highlights
- Rough surfaces
  - Micro-facet normals vary strongly
  - Blurry highlight

Polished
Smooth
Rough
Very rough
Glossy reflection

• Expect most light to be reflected in mirror direction

• Because of micro-facets, some light is reflected slightly off ideal reflection direction

• Reflection
  – Brightest when view vector is aligned with reflection
  – Decreases as angle between view vector and reflection direction increases
Phong reflectance model

Symmetric V-shaped grooves – 'microfacets'

Average surface normal

Phong Lobe
(Lobe illustrates brightness in a direction)
Phong Model

Mirror

Diffuse
General BRDF

Example: velvet

*Portrait of Sir Thomas Morre*, Hans Holbein the Younger, 1527
Shadows

- Give additional cues on scene lighting
Shadows

- Contact points
- Depth cues
Shadows cast by a point source

- A point that cannot see the source is in shadow
- For point sources, two types of shadows: cast shadows & attached shadows
**Terminology**

**Umbra**: fully shadowed region

**Penumbra**: partially shadowed region
Penumbra and Umbra
Hard and soft shadows

- Point and directional lights lead to hard shadows, no penumbra
- Area light sources lead to soft shadows, with penumbra
Hard and soft shadows

Hard shadow from point light source

Soft shadow from area light source
Next Lecture

• Photometric Stereo
• Reading:
  – Szeliski
    • Section 13.1.1