CSE 151: Introduction to Machine Learning	Spring 2018
Problem Set 5	
Instructor: Kamalika Chaudhuri	**Due on:** never

Problem 1: 8 points

In the following problems, suppose that K, K_1 and K_2 are kernels with feature maps ϕ, ϕ^1 and ϕ^2. For the following functions $K'(x, z)$, state if they are kernels or not. If they are kernels, write down the corresponding feature map, in terms of ϕ, ϕ^1, ϕ^2 and c, c_1, c_2. If they are not kernels, prove that they are not.

1. $K'(x, z) = cK(x, z)$, for $c > 0$.
2. $K'(x, z) = cK(x, z)$, where $c < 0$, and there exists some x for which $K(x, x) > 0$.
3. $K'(x, z) = c_1 K_1(x, z) + c_2 K_2(x, z)$ for $c_1, c_2 > 0$.
4. $K'(x, z) = K_1(x, z) K_2(x, z)$.

Problem 2: 14 points

For the following functions $K(x, z)$, state if it is a kernel or not. If the function is a kernel, then write down its feature map. If it is not a kernel, prove that it is not one. For your proof, you can use the answers to Problem 1.

1. $x = [x_1, x_2]$, $z = [z_1, z_2]$, x_1, x_2, z_1, z_2 are real numbers. $K(x, z) = x_1 z_2$.
2. Let $x = [x_1, \ldots, x_d]$, $z = [z_1, \ldots, z_d]$, x_is and z_is are real numbers. $K(x, z) = 1 - \langle x, z \rangle$.
3. $x = [x_1, \ldots, x_d]$, $z = [z_1, \ldots, z_d]$, and f is a function. $K(x, z) = f(x_1, x_2) f(z_1, z_2)$.
4. $x = [x_1, \ldots, x_d]$, $z = [z_1, \ldots, z_d]$, x_is and z_is are integers between 0 and 100. $K(x, z) = \sum_{i=1}^{d} \min(x_i, z_i)$.
5. $x = [x_1, \ldots, x_d]$, $z = [z_1, \ldots, z_d]$, x_is and z_is are real numbers.
$$K(x, z) = (1 + x_1 z_1)(1 + x_2 z_2) \ldots (1 + x_d z_d)$$
6. $x = [x_1, \ldots, x_d]$, $z = [z_1, \ldots, z_d]$, x_is and z_is are integers between 0 and 100. $K(x, z) = \sum_{i=1}^{d} \max(x_i, z_i)$.
7. x are z are documents with words from some dictionary D. $K(x, z)$ is the number of words that occur in both x and z, where each unique common word is counted once.