CSE 151: Introduction to Machine Learning		Winter 2018
	Problem Set 0	
Instructor: Kamalika Chaudhuri		**Due on:** never

Problem 1: (10 points)

Let X and Y be random variables with the following joint distribution:

	$X = 1$	$X = 2$	$X = 3$	$X = 4$
$Y = 0$	1/18	1/18	1/9	1/9
$Y = 1$	1/12	1/12	1/6	0
$Y = 2$	0	1/30	1/30	4/15

1. What are the marginal distributions of X and Y?
2. Are X and Y independent? Justify your answer.
3. What is the conditional distribution of X, given that $Y = 2$? What is $\mathbb{E}[X|Y = 2]$?
4. Calculate $\mathbb{E}[X]$, $\mathbb{E}[Y]$ and $\mathbb{E}[XY]$.

Problem 2: (4 points)

A coin is tossed three times with probability of heads p. Consider the following four events:

- A: Heads on the first toss
- B: Tails on the second toss
- C: All three outcomes the same
- D: Exactly one head

Which of the following pairs of events are independent? (More than one pair may be independent.) Justify your answer.
1. A and B 2. A and C 3. A and D 4. C and D

Problem 3: (6 points)

Let A and B be the following matrices, and let x be the row vector: $x = [10, 1, 1]$.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

1. Calculate $Ax^\top$ and $xB^\top$.
2. What is the determinant of A? What is the determinant of B?
3. Is $AB = BA$? Justify your answer.

Problem 4: (8 points)

Let $v_1 = [1, -1, 2, 0]$, $v_2 = [1, 0, 1, 1]$, $v_3 = [1, -2, 3, -1]$, and $v_4 = [3, 1, 2, 4]$.

1. Are v_1, v_2, v_3, v_4 linearly independent? Justify your answer.
2. Let U be the 4×4 matrix whose rows are $v_1, \ldots, v_4$. What is the rank of U? Justify your answer.
3. Write down a basis of the null-space of U and a basis of the range of U.

Problem 5: (3 points)

Suppose you have a deck of 52 cards, and you draw cards from the deck with replacement uniformly at random independently. Let $X_1, X_2, \ldots, X_{50}$ be the outcomes of the first 50 draws. Thus, each random variable X_i can take values $1, \ldots, 52$, and the probability that it takes each of these values is $\frac{1}{52}$.

1. What is $\mathbb{E}[X_1]$?
2. Let $Z = X_1 - 2X_2 + 3X_3$. What is $\mathbb{E}[Z]$?
3. Let $Y = X_1 - X_2 + X_3 - X_4 + \ldots + X_{49} - X_{50}$. What is $\mathbb{E}[Y]$?

Problem 6: (9 points)

Consider the following functions:

$$f_1(x) = e^{10x+2}, \; f_2(x) = 5x^{12} + 2, \; f_3(x) = \frac{1}{1-x}$$

1. Write down the derivatives of f_1, f_2 and f_3 with respect to x. Are any of these functions monotonically increasing for all x? Are any of them monotonically decreasing for all x?
2. Write down the integrals:
$$\int f_1(x)dx, \; \int f_2(x)dx, \; \int f_3(x)dx$$
3. Draw a graph of the implicit function $x^2 + 4y^2 = 4$. Clearly label the regions where $x^2 + 4y^2 < 4$ and where $x^2 + 4y^2 > 4$.