

-
- (1) This is an open book, open notes exam. You are free to consult any text book or notes. **You are not allowed to consult with any other person.**
 - (2) If you need any clarification, please post a private message to the instructors on Piazza.
 - (3) Remember that your work is graded on the *clarity* of your writing and explanation as well as the validity of what you write.
 - (4) This is a one-hour exam.
-
- (1) Draw the decision boundary for the nearest neighbor classifier on the following data points in 2 dimensions.

$$((0, 0), 0), ((4, 1), 1), ((-2, 3), 0)$$

For full credit, write down the equation for each segment of the decision boundary, and label each region with the label assigned to it by the classifier.

- (2) Remember that any classifier is basically a function that takes in a feature vector in \mathbb{R}^d and outputs a label. We say that two classifiers C and C' are equal if they output the same label for every feature vector x in \mathbb{R}^d . Formally, for all $x \in \mathbb{R}^d$, $C(x) = C'(x)$.

Suppose Alice and Bob are both building a 1-nearest neighbor classifier on the same training dataset S . To build the classifier Alice computes the nearest neighbor using the Euclidean distance to get the classifier C .

- (a) (5 points) Suppose Bob computes the nearest neighbors using the square of the Euclidean distance and gets the classifier C' . Is C' equal to C for all training datasets? If yes, provide a short proof, and if no, provide a counterexample.

- (b) (5 points) Now, instead of the square of the Euclidean distance, Bob computes the nearest neighbors using the following distance function:

$$d(x, x') = \sum_{i=1}^d (x^i - x'^i)^3$$

where x^i denotes coordinate i of the vector x . This gives him a classifier C'' . Is C'' equal to C for all training datasets? Justify your answer.