CSE 151A Winter 2021 - Take Home Exam 1

1 hour **Name:** **PID:**

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(1) This is an open book, open notes exam. You are free to consult any text book or notes. **You are not allowed to consult with any other person.**

(2) If you need any clarification, please post a private message to the instructors on Piazza.

(3) Remember that your work is graded on the *clarity* of your writing and explanation as well as the validity of what you write.

(4) This is a one-hour exam.

(1) In class we defined positive semidefinite (PSD) matrices. A $d \times d$ matrix A is said to be positive definite (PD) if for all $d \times 1$ vectors x that are not the all-zeros vector, we have that $x^\top A x > 0$. Observe that a positive definite matrix is also positive semi-definite, but a positive semi-definite matrix may not be positive definite.

For each of the following matrices, state if they are (a) positive definite (b) positive semi-definite but not positive definite (c) not positive semi-definite. Justify your answer with a short proof.

(a) (3 points) Let u be a $d \times 1$ unit vector – that is, $\|u\| = 1$. $A = uu^\top$.

(b) (3 points) $B = 2I_d$. (Recall I_d is the $d \times d$ identity matrix.)

(c) (4 points) Let u be a $d \times 1$ unit vector – that is, $\|u\| = 1$. $C = uu^\top - \frac{1}{2}I_d$.

(2) (5 points) Let x be a $d \times 1$ vector, let $z_1, \ldots, z_n$ be n vectors, where each z_i is $d \times 1$, and let $f : \mathbb{R}^d \to \mathbb{R}$ be the following multivariate function of x:

$$f(x) = \sum_{i=1}^{n} e^{x^\top z_i}$$

Write down the gradient of f with respect to x. Write down an algorithm that computes this gradient in $O(nd)$ time when given the vectors $z_1, \ldots, z_n$.

(3) (5 points) Given a $d \times d$ matrix A, its Frobenius norm is defined as:

$$\|A\|_F = \sqrt{\sum_{i=1}^{d} \sum_{j=1}^{d} A_{ij}^2}$$

Now suppose we are given two $d \times 1$ vectors x and y. State whether the following statement is true or false. Justify your answer.

$$x^\top y \leq \|xy^\top\|_F$$