(1) This is an open book, open notes exam. You are free to consult any text book or notes. **You are not allowed to consult with any other person.**

(2) If you need any clarification, please post a private message to the instructors on Piazza.

(3) Remember that your work is graded on the **clarity** of your writing and explanation as well as the validity of what you write.

(4) This is a one-hour exam.

(1) Let $X$ and $Y$ be random variables with the following joint distribution.

<table>
<thead>
<tr>
<th></th>
<th>$X = 1$</th>
<th>$X = 2$</th>
<th>$X = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y = 0$</td>
<td>$1/4$</td>
<td>$1/4$</td>
<td>$1/4$</td>
</tr>
<tr>
<td>$Y = 1$</td>
<td>$1/12$</td>
<td>$0$</td>
<td>$1/6$</td>
</tr>
</tbody>
</table>

Now answer the following questions.

(a) (4 points) What are the marginal distributions of $X$ and $Y$?

(b) (4 points) What are the conditional expectations $E[X|Y = 1]$ and $E[Y|X = 1]$?

(c) (2 points) Are $X$ and $Y$ independent? Justify your answer.
(2) (5 points) Let $a_1, \ldots, a_k$ be $k$ real numbers. Consider the following function:

$$f(x) = \sum_{i=1}^{k} \log(1 + e^{a_i x})$$

Write down the derivative of $f(x)$. Use this derivative to write down a condition on the numbers $a_1, \ldots, a_k$ that ensures that $f(x)$ is strictly increasing at $x = 0$.

(3) (5 points) Let $v_1 = [3, 5, 1], v_2 = [2, 1, 3] \text{ and } v_3 = [12, 13, 11]$. Are $v_1, v_2 \text{ and } v_3$ linearly independent? Justify your answer.