- (1) This is an open book, open notes exam. You are free to consult any text book or notes. You are not allowed to consult with any other person.
- (2) If you need any clarification, please post a private message to the instructors on Piazza.
- (3) Remember that your work is graded on the *clarity* of your writing and explanation as well as the validity of what you write.
- (4) This is a one-hour exam.
- (1) Let X and Y be random variables with the following joint distribution.

	X = 1	X = 2	X = 3
Y = 0	1/4	1/4	1/4
Y = 1	1/12	0	1/6

Now answer the following questions.

(a) (4 points) What are the marginal distributions of X and Y?

(b) (4 points) What are the conditional expectations  $\mathbb{E}[X|Y=1]$  and  $\mathbb{E}[Y|X=1]$ ?

(c) (2 points) Are X and Y independent? Justify your answer.

(2) (5 points) Let  $a_1, \ldots, a_k$  be k real numbers. Consider the following function:

$$f(x) = \sum_{i=1}^{k} \log(1 + e^{a_i x})$$

Write down the derivative of f(x). Use this derivative to write down a condition on the numbers  $a_1, \ldots, a_k$  that ensures that f(x) is strictly increasing at x = 0.

(3) (5 points) Let  $v_1 = [3, 5, 1]$ ,  $v_2 = [2, 1, 3]$  and  $v_3 = [12, 13, 11]$ . Are  $v_1$ ,  $v_2$  and  $v_3$  linearly independent? Justify your answer.