

HW3

February 5, 2020

1 CSE 252B: Computer Vision II, Winter 2019 – Assignment 3

1.0.1 Instructor: Ben Ochoa

1.0.2 Due: Wednesday, February 19, 2020, 11:59 PM

1.1 Instructions

- Review the academic integrity and collaboration policies on the course website.
- This assignment must be completed individually.
- All solutions must be written in this notebook
- Math problems must be done in Markdown/LATEX.
- You must show your work and describe your solution.
- Programming aspects of this assignment must be completed using Python in this notebook.
- Your code should be well written with sufficient comments to understand, but there is no need to write extra markdown to describe your solution if it is not explicitly asked for.
- This notebook contains skeleton code, which should not be modified (This is important for standardization to facilitate efficient grading).
- You may use python packages for basic linear algebra, but you may not use packages that directly solve the problem. If you are uncertain about using a specific package, then please ask the instructional staff whether or not it is allowable.
- You must submit this notebook exported as a pdf. You must also submit this notebook as an .ipynb file.
- Your code and results should remain inline in the pdf (Do not move your code to an appendix).
- You must submit both files (.pdf and .ipynb) on Gradescope. You must mark each problem on Gradescope in the pdf.
- It is highly recommended that you begin working on this assignment early.

1.2 Problem 1 (Programming): Estimation of the Camera Pose - Outlier rejection (20 points)

Download input data from the course website. The file `hw3_points3D.txt` contains the coordinates of 60 scene points in 3D (each line of the file gives the \tilde{X}_i , \tilde{Y}_i , and \tilde{Z}_i inhomogeneous coordinates of a point). The file `hw3_points2D.txt` contains the coordinates of the 60 corresponding image points in 2D (each line of the file gives the \tilde{x}_i and \tilde{y}_i inhomogeneous coordinates of a point). The corresponding 3D scene and 2D image points contain both inlier and outlier correspondences. For the inlier correspondences, the scene points have been randomly generated and projected to

image points under a camera projection matrix (i.e., $x_i = PX_i$), then noise has been added to the image point coordinates.

The camera calibration matrix was calculated for a 1280×720 sensor and 45° horizontal field of view lens. The resulting camera calibration matrix is given by

$$K = \begin{bmatrix} 1545.0966799187809 & 0 & 639.5 \\ 0 & 1545.0966799187809 & 359.5 \\ 0 & 0 & 1 \end{bmatrix}$$

For each image point $x = (x, y, w)^\top = (\tilde{x}, \tilde{y}, 1)^\top$, calculate the point in normalized coordinates $\hat{x} = K^{-1}x$.

Determine the set of inlier point correspondences using the M-estimator Sample Consensus (MSAC) algorithm, where the maximum number of attempts to find a consensus set is determined adaptively. For each trial, use the 3-point algorithm of Finsterwalder (as described in the paper by Haralick et al.) to estimate the camera pose (i.e., the rotation R and translation t from the world coordinate frame to the camera coordinate frame), resulting in up to 4 solutions, and calculate the error and cost for each solution. Note that the 3-point algorithm requires the 2D points in normalized coordinates, not in image coordinates. Calculate the projection error, which is the (squared) distance between projected points (the points in 3D projected under the normalized camera projection matrix $\hat{P} = [R|t]$) and the measured points in normalized coordinates (hint: the error tolerance is simpler to calculate in image coordinates using $P = K[R|t]$ than in normalized coordinates using $\hat{P} = [R|t]$).

Hint: this problem has codimension 2.

Report your values for:

- the probability p that as least one of the random samples does not contain any outliers
- the probability α that a given point is an inlier
- the resulting number of inliers
- the number of attempts to find the consensus set

```
In [1]: import numpy as np
import time

def Homogenize(x):
    # converts points from inhomogeneous to homogeneous coordinates
    return np.vstack((x,np.ones((1,x.shape[1]))))

def Dehomogenize(x):
    # converts points from homogeneous to inhomogeneous coordinates
    return x[:-1]/x[-1]

# load data
x0=np.loadtxt('hw3_points2D.txt').T
X0=np.loadtxt('hw3_points3D.txt').T
print('x is', x0.shape)
print('X is', X0.shape)

K = np.array([[1545.0966799187809, 0, 639.5],
```

```

        [0, 1545.0966799187809, 359.5],
        [0, 0, 1]])

print('K =')
print(K)

def ComputeCost(P, x, X, K):
    # Inputs:
    #   P - camera projection matrix
    #   x - 2D groundtruth image points
    #   X - 3D groundtruth scene points
    #   K - camera calibration matrix
    #
    # Output:
    #   cost - total projection error
    n = x.shape[1]
    covarx = np.eye(2*n) # covariance propagation

    """your code here"""

    cost = np.inf
    return cost

x is (2, 60)
X is (3, 60)
K =
[[1.54509668e+03 0.00000000e+00 6.39500000e+02]
 [0.00000000e+00 1.54509668e+03 3.59500000e+02]
 [0.00000000e+00 0.00000000e+00 1.00000000e+00]]

In [2]: from scipy.stats import chi2

def MSAC(x, X, K, thresh, tol, p):
    # Inputs:
    #   x - 2D inhomogeneous image points
    #   X - 3D inhomogeneous scene points
    #   K - camera calibration matrix
    #   thresh - cost threshold
    #   tol - reprojection error tolerance
    #   p - probability that as least one of the random samples does not contain any ou
    #
    # Output:
    #   consensus_min_cost - final cost from MSAC
    #   consensus_min_cost_model - camera projection matrix P
    #   inliers - list of indices of the inliers corresponding to input data

```

```

# trials - number of attempts taken to find consensus set

"""your code here"""

trials = 0
max_trials = np.inf
consensus_min_cost = np.inf
consensus_min_cost_model = np.zeros((3,4))
inliers = np.random.randint(0, 59, size=10)
return consensus_min_cost, consensus_min_cost_model, inliers, trials

# MSAC parameters
thresh = 100
tol = 0
p = 0
alpha = 0

tic=time.time()

cost_MSAC, P_MSAC, inliers, trials = MSAC(x0, X0, K, thresh, tol, p)

# choose just the inliers
x = x0[:,inliers]
X = X0[:,inliers]

toc=time.time()
time_total=toc-tic

# display the results
print('took %f secs'%time_total)
# print('%d iterations'%trials)
# print('inlier count: ',len(inliers))
print('MSAC Cost=%.9f'%cost_MSAC)
print('P = ')
print(P_MSAC)
print('inliers: ',inliers)

took 0.000955 secs
MSAC Cost=inf
P =
[[0. 0. 0. 0.]
 [0. 0. 0. 0.]
 [0. 0. 0. 0.]]
inliers: [58 26 14 32 38 37 40 33 31 1]

```

Final values for parameters

- $p =$
- $\alpha =$
- tolerance =
- num_inliers =
- num_attempts =

1.3 Problem 2 (Programming): Estimation of the Camera Pose - Linear Estimate (30 points)

Estimate the normalized camera projection matrix $\hat{P}_{\text{linear}} = [R_{\text{linear}} | t_{\text{linear}}]$ from the resulting set of inlier correspondences using the linear estimation method (based on the EPnP method) described in lecture. Report the resulting R_{linear} and t_{linear} .

```
In [3]: def EPnP(x, X, K):
        # Inputs:
        #   x - 2D inlier points
        #   X - 3D inlier points
        # Output:
        #   P - normalized camera projection matrix

        """your code here"""

        R = np.eye(3)
        t = np.array([[1,0,0]]).T
        P = np.concatenate((R, t), axis=1)
        return P

        # Uncomment the following block to run EPnP on an example set of inliers.
        # You MUST comment it before submitting, or you will lose points.
        # The sample cost with these inliers should be around 57.85.
        # inliers = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24]
        # x = x0[:,inliers]
        # X = X0[:,inliers]

        tic=time.time()
        P_linear = EPnP(x, X, K)
        toc=time.time()
        time_total=toc-tic

        # display the results
        print('took %f secs'%time_total)
        print('R_linear = ')
        print(P_linear[:,0:3])
        print('t_linear = ')
        print(P_linear[:, -1])
```

```

took 0.000115 secs
R_linear =
[[1. 0. 0.]
 [0. 1. 0.]
 [0. 0. 1.]]
t_linear =
[1. 0. 0.]

```

1.4 Problem 3 (Programming): Estimation of the Camera Pose - Nonlinear Estimate (30 points)

Use R_{linear} and t_{linear} as an initial estimate to an iterative estimation method, specifically the Levenberg-Marquardt algorithm, to determine the Maximum Likelihood estimate of the camera pose that minimizes the projection error under the normalized camera projection matrix $\hat{P} = [R|t]$. You must parameterize the camera rotation using the angle-axis representation ω (where $[\omega]_{\times} = \ln R$) of a 3D rotation, which is a 3-vector.

Report the initial cost (i.e. cost at iteration 0) and the cost at the end of each successive iteration. Show the numerical values for the final estimate of the camera rotation ω_{LM} and R_{LM} , and the camera translation t_{LM} .

```

In [4]: from scipy.linalg import block_diag

# Note that np.sinc is different than defined in class
def Sinc(x):
    """your code here"""

    y = x
    return y

def skew(w):
    # Returns the skew-symmetric representation of a vector
    """your code here"""

    return w_skew

def Parameterize(R):
    # Parameterizes rotation matrix into its axis-angle representation
    """your code here"""

    w = np.array([[1,0,0]]).T
    theta = 0
    return w, theta

```

```

def Deparameterize(w):
    # Deparameterizes to get rotation matrix
    """your code here"""

    return R

def Jacobian(R, w, t, X):
    # compute the jacobian matrix
    # Inputs:
    #   R - 3x3 rotation matrix
    #   w - 3x1 axis-angle parameterization of R
    #   t - 3x1 translation vector
    #   X - 3D inlier points
    #
    # Output:
    #   J - Jacobian matrix of size 2*nx6

    """your code here"""

    J = np.zeros((2*X.shape[1],6))
    return J

```

```

In [5]: def LM(P, x, X, K, max_iters, lam):
    # Inputs:
    #   P - initial estimate of camera pose
    #   x - 2D inliers
    #   X - 3D inliers
    #   K - camera calibration matrix
    #   max_iters - maximum number of iterations
    #   lam - lambda parameter
    #
    # Output:
    #   P - Final camera pose obtained after convergence

    """your code here"""

    for i in range(max_iters):

        cost = ComputeCost(P, x, X, K)
        print('iter %03d Cost %.9f'%(i+1, cost))

    return P

```

```

# With the sample inliers...
# Start: 57.854356308
# End: 57.815478730

# LM hyperparameters
lam = .001
max_iters = 100

tic = time.time()
P_LM = LM(P_linear, x, X, K, max_iters, lam)
w_LM,_ = Parameterize(P_LM[:,0:3])
toc = time.time()
time_total = toc-tic

# display the results
print('took %f secs'%time_total)
print('w_LM = ')
print(w_LM)
print('R_LM = ')
print(P_LM[:,0:3])
print('t_LM = ')
print(P_LM[:,-1])

```

```

iter 001 Cost inf
iter 002 Cost inf
iter 003 Cost inf
iter 004 Cost inf
iter 005 Cost inf
iter 006 Cost inf
iter 007 Cost inf
iter 008 Cost inf
iter 009 Cost inf
iter 010 Cost inf
iter 011 Cost inf
iter 012 Cost inf
iter 013 Cost inf
iter 014 Cost inf
iter 015 Cost inf
iter 016 Cost inf
iter 017 Cost inf
iter 018 Cost inf
iter 019 Cost inf
iter 020 Cost inf
iter 021 Cost inf
iter 022 Cost inf

```


iter 023 Cost inf
iter 024 Cost inf
iter 025 Cost inf
iter 026 Cost inf
iter 027 Cost inf
iter 028 Cost inf
iter 029 Cost inf
iter 030 Cost inf
iter 031 Cost inf
iter 032 Cost inf
iter 033 Cost inf
iter 034 Cost inf
iter 035 Cost inf
iter 036 Cost inf
iter 037 Cost inf
iter 038 Cost inf
iter 039 Cost inf
iter 040 Cost inf
iter 041 Cost inf
iter 042 Cost inf
iter 043 Cost inf
iter 044 Cost inf
iter 045 Cost inf
iter 046 Cost inf
iter 047 Cost inf
iter 048 Cost inf
iter 049 Cost inf
iter 050 Cost inf
iter 051 Cost inf
iter 052 Cost inf
iter 053 Cost inf
iter 054 Cost inf
iter 055 Cost inf
iter 056 Cost inf
iter 057 Cost inf
iter 058 Cost inf
iter 059 Cost inf
iter 060 Cost inf
iter 061 Cost inf
iter 062 Cost inf
iter 063 Cost inf
iter 064 Cost inf
iter 065 Cost inf
iter 066 Cost inf
iter 067 Cost inf
iter 068 Cost inf
iter 069 Cost inf
iter 070 Cost inf

```
iter 071 Cost inf
iter 072 Cost inf
iter 073 Cost inf
iter 074 Cost inf
iter 075 Cost inf
iter 076 Cost inf
iter 077 Cost inf
iter 078 Cost inf
iter 079 Cost inf
iter 080 Cost inf
iter 081 Cost inf
iter 082 Cost inf
iter 083 Cost inf
iter 084 Cost inf
iter 085 Cost inf
iter 086 Cost inf
iter 087 Cost inf
iter 088 Cost inf
iter 089 Cost inf
iter 090 Cost inf
iter 091 Cost inf
iter 092 Cost inf
iter 093 Cost inf
iter 094 Cost inf
iter 095 Cost inf
iter 096 Cost inf
iter 097 Cost inf
iter 098 Cost inf
iter 099 Cost inf
iter 100 Cost inf
took 0.008562 secs
w_LM =
[[1]
 [0]
 [0]]
R_LM =
[[1. 0. 0.]
 [0. 1. 0.]
 [0. 0. 1.]]
t_LM =
[1. 0. 0.]
```