Probability review I:
Probability spaces, events, and conditioning

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Probability spaces

You roll two dice.

What is the probability they add to 10?

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The **probability space** has two components:

1. **Sample space** (space of outcomes).

2. **Probabilities of outcomes**, summing to 1.

Events

Probability space:

- Outcomes: $\Omega = \{\text{all possible pairs of dice rolls}\}$
- Every pair $z = (z_1, z_2) \in \Omega$ has probability $1/36$.

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Event of interest: the two dice add up to 10.

Another example

A drawer has 3 blue socks and 3 red socks.
You pull out two socks at random.
What is the probability they match?

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Outcome space $\Omega = \{(b, b), (b, r), (r, b), (r, r)\}$.

Event of interest: $A = \{(b, b), (r, r)\}$.

Multiple events

Once again, roll two dice.

- Outcome space $\Omega = \{1, 2, 3, 4, 5, 6\}^2$
- All outcomes equally likely: probability $1/36$

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Two events of interest:

- $A =$ first roll is a four
- $B =$ the sum is ten

Event that they **both** occur: $A \cap B$

Conditioning

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Conditional probability

$\Pr(B|A) =$ probability that B occurs, given that A occurs

The conditioning formula

Roll two dice.

- Event A: first roll is a four
- Event B: sum is ten

Formula: $\boxed{\Pr(A \cap B) = \Pr(A)\Pr(B|A)}$

Bayes' rule

Roll two dice.

- Event A: first roll is a four. $\Pr(A) = 1/6$.
- Event B: sum is ten. $\Pr(B) = 1/12$.
- Conditional probability $\Pr(B|A) = 1/6$.

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Bayes' rule: $\Pr(A|B) = \Pr(A) \times \dfrac{\Pr(B|A)}{\Pr(B)}$