CSE 202 Homework 2
Spring, 2018
Path search, using and modifying algorithms, fitting data structures to algorithms. All parts are worth 20 points. Due Monday, January 29

Exercises

Even length paths Give an efficient algorithm to determine the set of vertices in a directed graph that can be reached from s via a path of even length

Round trip times The round trip distance between s and t in a strongly connected weighted directed graph is the minimum total lengths of edges of a cycle that contains both s and t. Give as efficient an algorithm as possible to, given s and t, compute the round trip distance between them.

Nearest k points Give an $O(k|V|)$ time algorithm that given a vertex s in a directed weighted graph, finds the k points that are closest to s.

Finding the largest k values in an array Give an $O(n \log k)$ time algorithm to find the largest k values in an unsorted array.

Problems

Roleplaying game You are playing a role-playing video game, where your character can go on a series of quests. There are n possible quests, and you have a fixed starting quest, and a fixed concluding quest. For each quest, you know the set of quests that can be attempted after finishing that quest; we can view this as a directed graph between quests. Your character has M hit points, and, for each quest , you know the number of hit points your character will lose (or, occasionally, gain through rest or healing potions) on that quest. Your character can never have more than M hit points (amounts gained more than that are wasted) and if your character goes to 0 or fewer hit points, the character dies and the game ends. You want to find a series of quests that your character survives and finishes the concluding quest. Give an efficient algorithm to find such a series or conclude that it is impossible. Analyse your algorithm in terms of both n, m the number of edges in the graph, and M.

Matrix sizes You can only multiply $r_1 \times c_1$ and $r_2 \times c_2$ dimensional matrices if $c_1 = r_2$, and the result is a $r_1 \times c_2$ dimensional matrix. Say that you are given a list of pairs of integers representing dimensions of matrix variables $M_1, ...M_n$, where M_i is a $r_i \times c_i$ dimensional matrix. You want to find a list of all possible dimensions (r, c) of products of sequences of matrices from among $M_1...M_n$. Give an efficient algorithm for this problem. (10 points correct algorithm, 10 points efficiency).

Flight scheduling You are devising a flight scheduler for a travel agency. The scheduler will get a list of available flights, and the customer's origin and destination. For each flight, it is given the cities and times of departure and arrival. The scheduler should output a list of flights that will take the customer from her origin to her destination that arrives as early as possible, subject to giving her at least 15 minutes for each connection. Give a formal specification for this problem (Instance, Solution Space, Constraints, Objective). (5 points) Then give as efficient as possible an algorithm to solve the problem. (5 points correct algorithm, 10 points efficiency).

Top k elements in heap: 20 pts Give an efficient algorithm that, given a binary max-heap H of size n and a number $1 \leq k \leq n$, returns the k largest elements of H. Analyze the time in terms of both k and n (although, for the best algorithm I know, the time does not depend on n at all, just k.)

(5 points correct algorithm, 15 points efficiency).

Implementation problem: arrays vs heaps in Dijkstra's algorithm Consider Dijkstra's algorithm for random directed graphs where each edge is present with probability p, and edges have weights that are uniform reals between 0 and 1. For a wide variety of numbers of vertices n, experimentally determine the value of p where you should switch from the array implementation to the heap implementation. How does p grow with n? Be sure to present your data clearly.