Toon Shading and Surface Patches

Computer Graphics

CSE 167

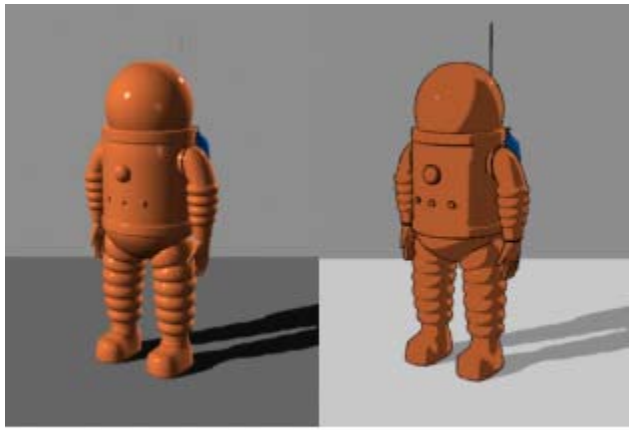
Lecture 14

CSE 167: Computer graphics

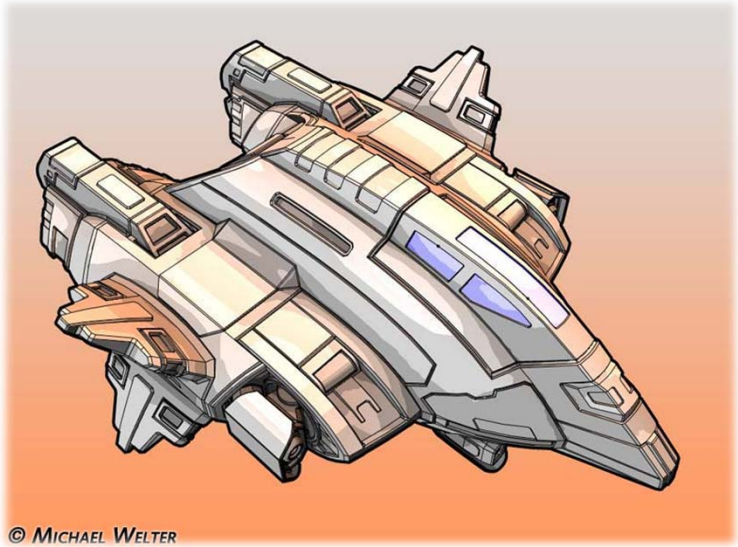
- Toon shading
  - A non-photorealistic rendering method
- Surface patches
  - Generalization of curves

Toon shading

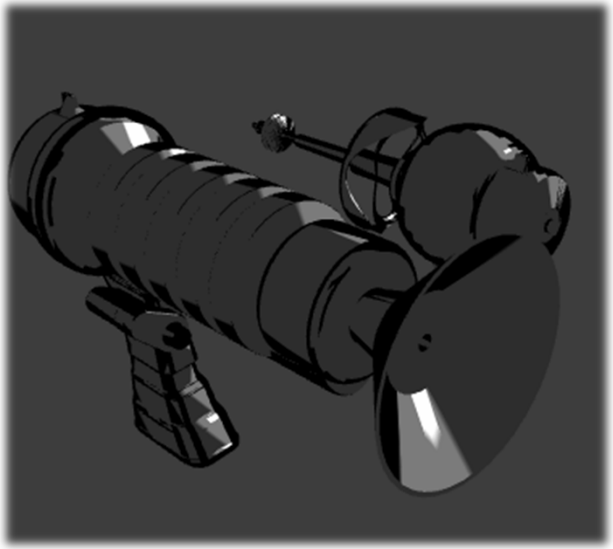
- Also called Cel Shading ("Cel" is short for "celluloid" sheets, on which animation was hand-drawn)
- Gives any 3D model a cartoon-style look
- Emphasizes silhouettes
- Discrete steps for diffuse shading, highlights
- Non-photorealistic rendering method (NPR)
- Programmable shaders allow real-time performance



Source: Wikipedia



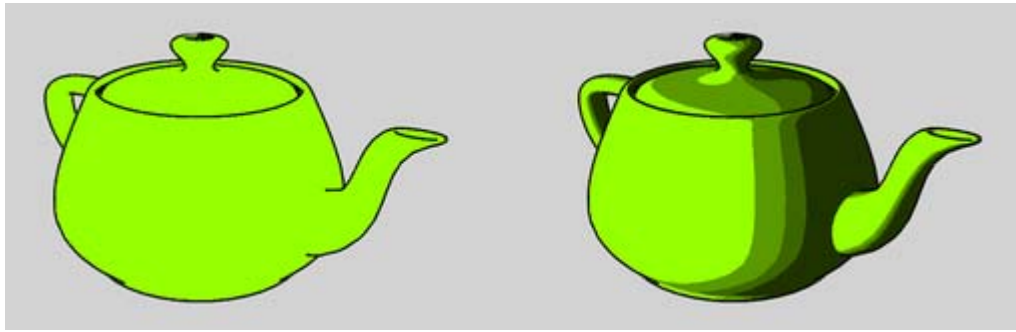
Off-line toon shader



GLSL toon shader

Approach

- Start with regular 3D model
- Apply two rendering tricks
  - Silhouette edges
    - Emphasize pixels with normals perpendicular to viewing direction
  - Discretized shading
    - Conventional (smooth) lighting values calculated for each pixel, then mapped to a small number of discrete shades



Source: Wikipedia

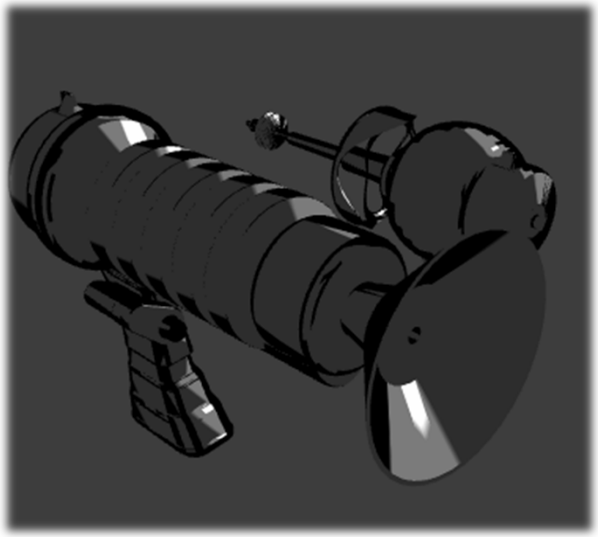
Silhouette edges

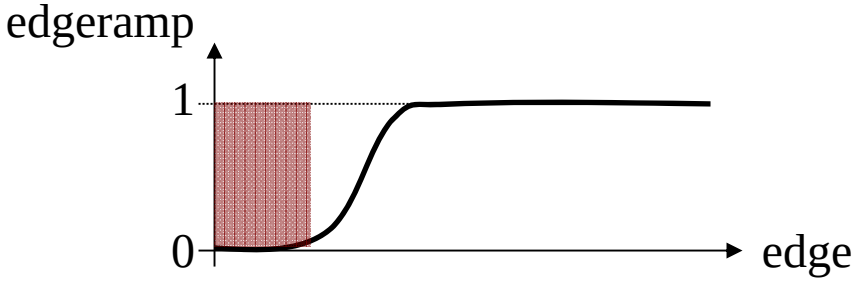
- Silhouette edge detection
  - Compute dot product of viewing direction **v** and normal **n**

$$\text{edge} = \max(0, \mathbf{n} \cdot \mathbf{v})$$

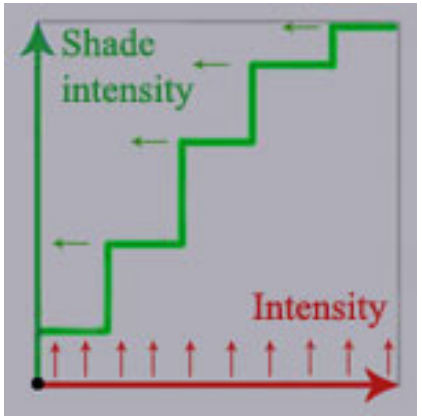
  - Use cutoff value for edge:
    - If edge < 0.01 draw black, else do not change pixel color
  - Use 1D texture to define edge ramp for smoother transition

```
uniform sample1D edgeramp;
e = texture1D(edgeramp,edge);
```





Discretized shading



- Compute diffuse and specular shading

$$\text{diffuse} = \mathbf{n} \cdot \mathbf{L} \quad \text{specular} = (\mathbf{n} \cdot \mathbf{h})^s$$

- Discretize shading
  - Approaches
    - If then else tree comparing values against thresholds

```
if (diffuse < A) diffuse = 0.0;
else if (diffuse < B) diffuse = B;
else if (diffuse < C) diffuse = C;
else diffuse = D;
```

    - 1D textures to map diffuse and specular shading to colors

```
uniform sampler1D diffuseramp;
uniform sampler1D specularramp;
color = e * (texture1D(diffuse,diffuseramp) +
texture1D(specular,specularramp));
```

Toon shading demo

http://www.bonzaisoftware.com/gldemos/non-photorealistic-rendering/

Surface patches

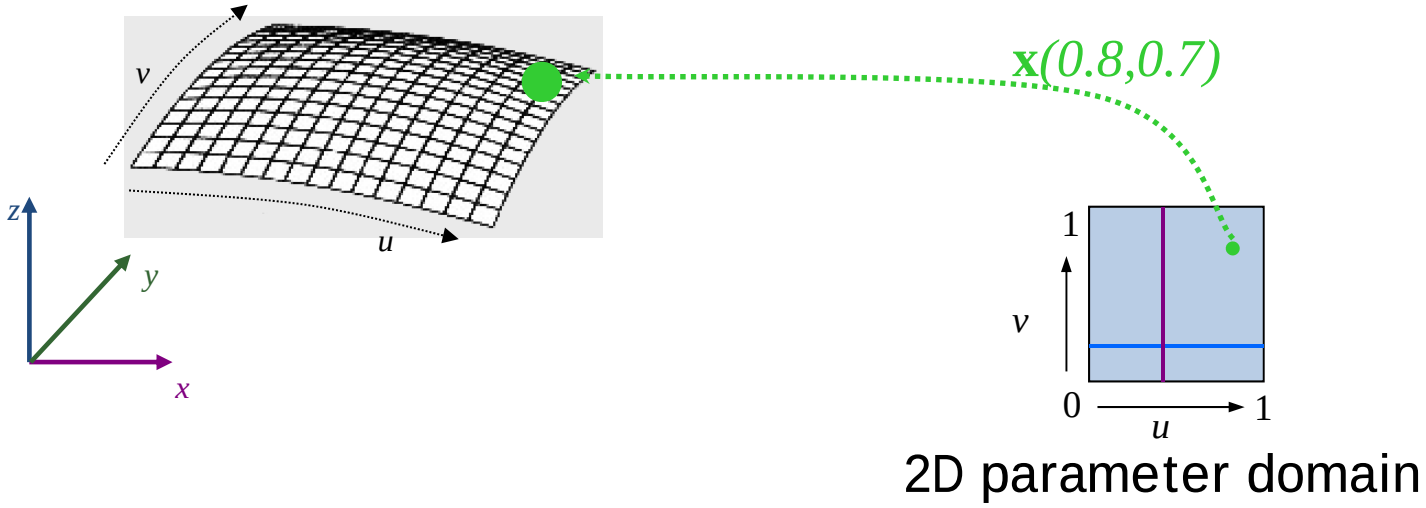
- Bilinear patch
- Bicubic Bézier patch

Curved surfaces

- Curves
  - Described by a 1D series of control points
  - A function $\mathbf{x}(t)$
  - Segments joined together to form a longer curve
- Surfaces
  - Described by a 2D mesh of control points
  - Parameters have two dimensions (two dimensional parameter domain)
  - A function $\mathbf{x}(u,v)$
  - Patches joined together to form a bigger surface

Parametric surface patch

- **x**(*u*,*v*) describes a point in space for any given (*u*,*v*) pair
  - *u*,*v* each range from 0 to 1



Parametric surface patch

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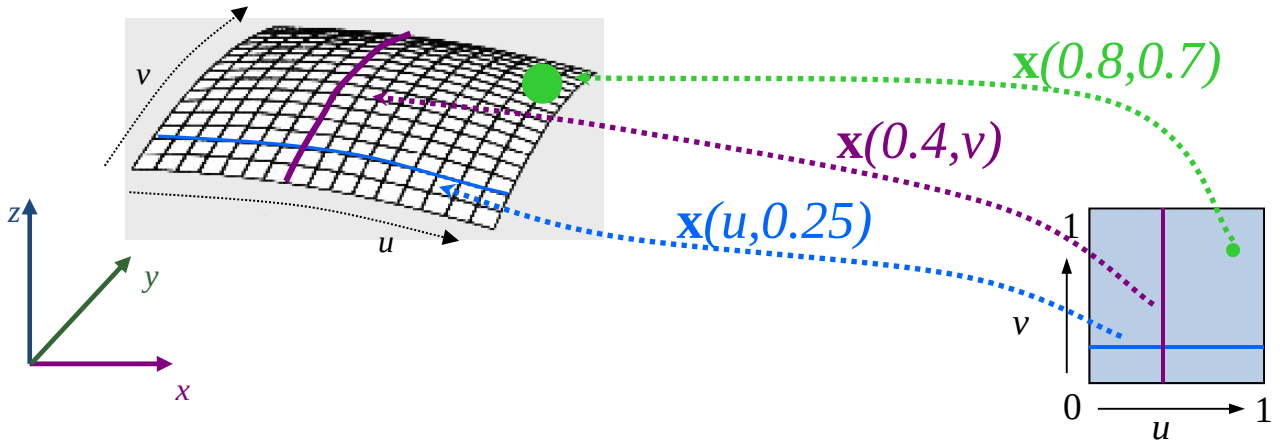
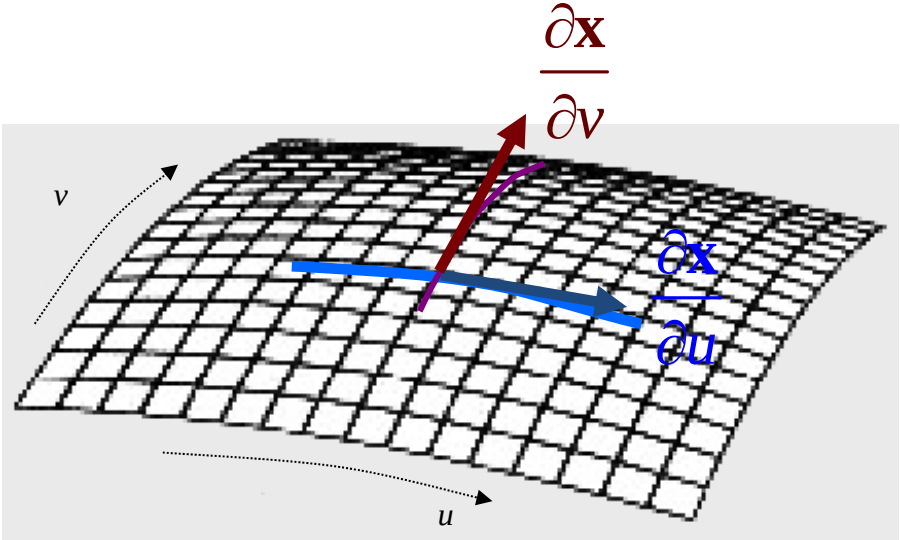


- Parametric curves
  - For fixed u_0 , have a *v* curve **x**(u_0,*v*)
  - For fixed v_0 , have a *u* curve **x**(*u*,v_0)
  - For any point on the surface, there are a pair of parametric curves through that point

Tangents

- The tangent to a parametric curve is also tangent to the surface
- For any point on the surface, there are a pair of (parametric) tangent vectors
- Note: these vectors are not necessarily perpendicular to each other



Surface normal

- Normal is cross product of the two tangent vectors

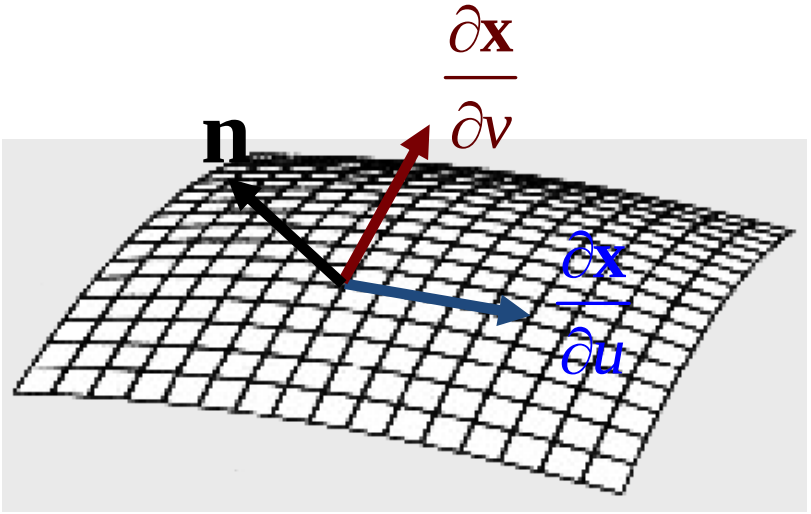
$$\mathbf{n}(u,v) = \frac{\partial \mathbf{x}}{\partial u}(u,v) \times \frac{\partial \mathbf{x}}{\partial v}(u,v)$$

Note: order matters

- Typically, we desire a unit normal, so we need to unitize

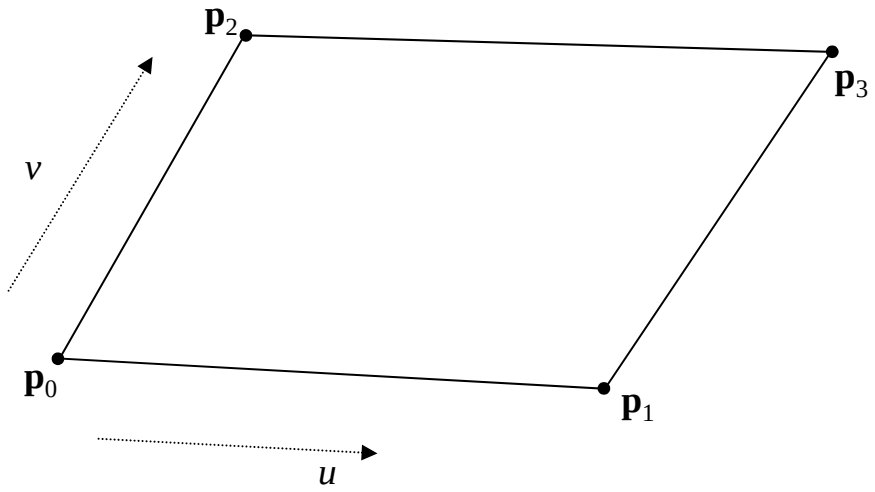
$$\mathbf{n}^*(u,v) = \frac{\partial \mathbf{x}}{\partial u}(u,v) \times \frac{\partial \mathbf{x}}{\partial v}(u,v)$$

$$\mathbf{n}(u,v) = \frac{\mathbf{n}^*(u,v)}{\left|\mathbf{n}^*(u,v)\right|}$$



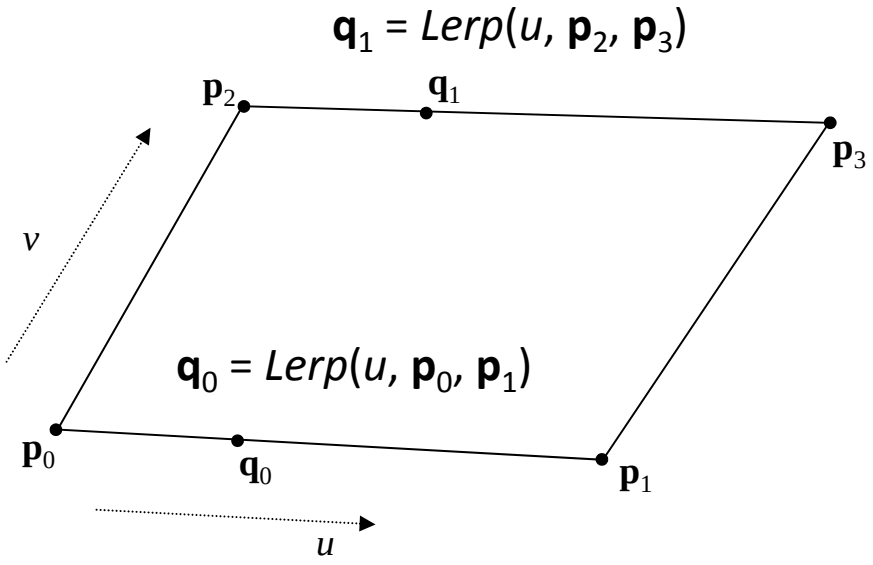
Bilinear patch

- Control mesh with four points $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$
- Compute $\mathbf{x}(u,v)$ using a two-step construction scheme



Bilinear patch (step 1)

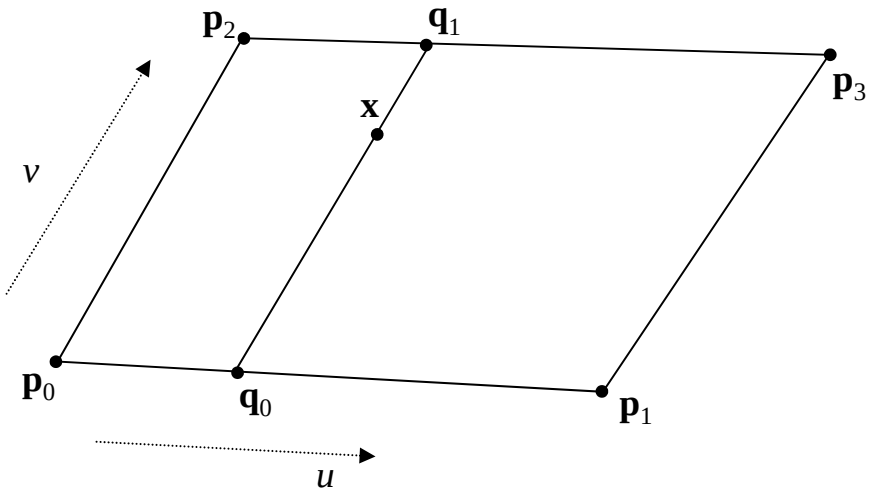
- For a given value of *u*, evaluate the linear curves on the two *u*-direction edges
- Use the same value *u* for both



Bilinear patch (step 2)

- Consider that $\mathbf{q}_0$, $\mathbf{q}_1$ define a line segment
- Evaluate it using v to get $\mathbf{x}$

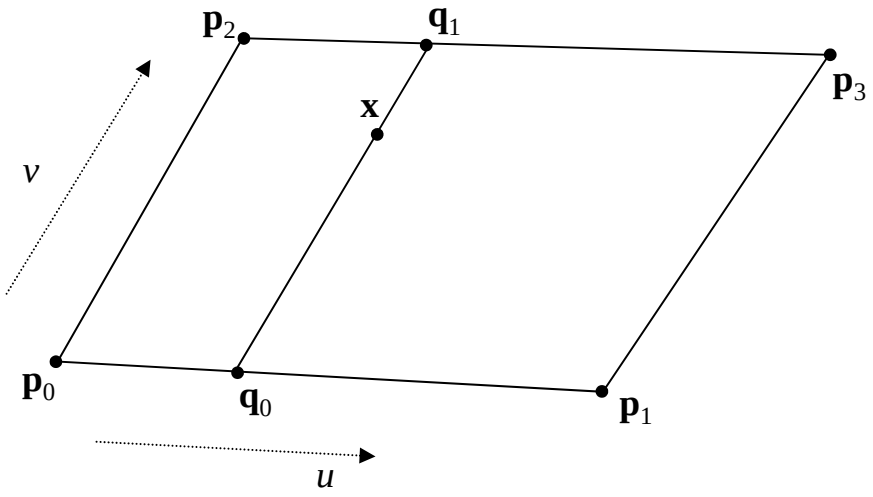
$$\mathbf{x} = Lerp(v, \mathbf{q}_0, \mathbf{q}_1)$$



Bilinear patch

- Combining the steps, we get

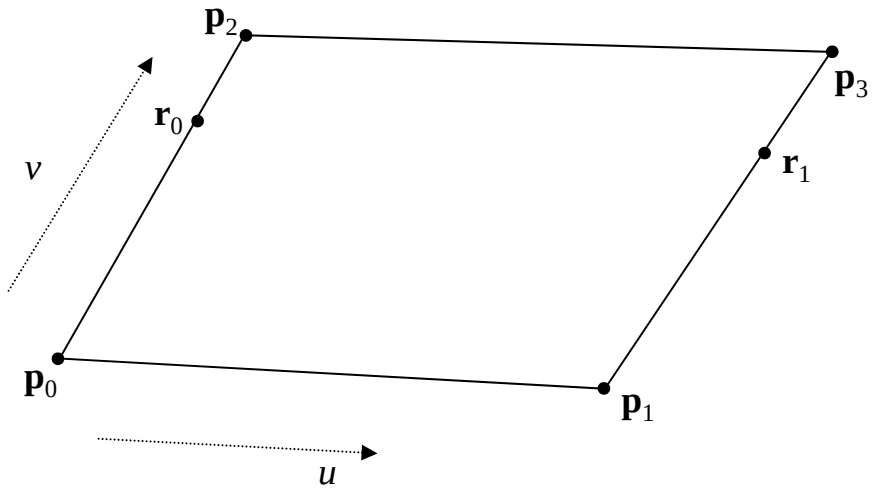
$$\mathbf{x}(u,v) = Lerp(v, Lerp(u, \mathbf{p}_0, \mathbf{p}_1), Lerp(u, \mathbf{p}_2, \mathbf{p}_3))$$



Bilinear patch

- Try the other order
- Evaluate first in the *v* direction

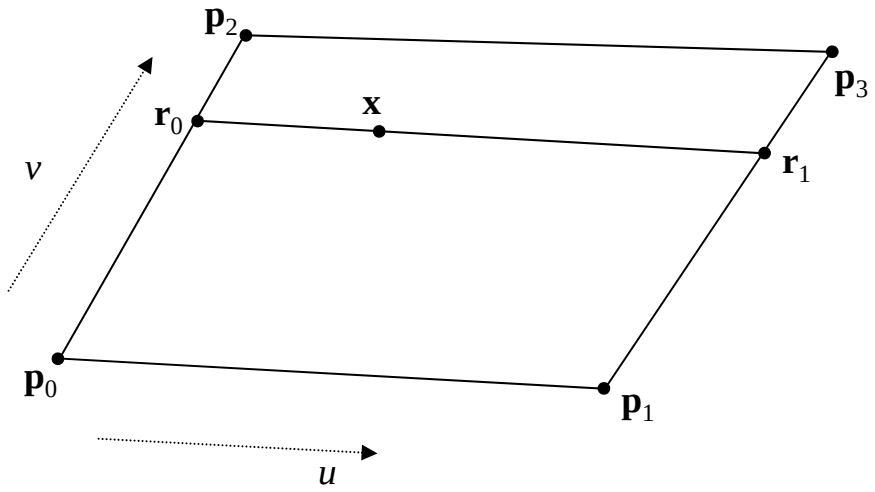
$$\mathbf{r}_0 = Lerp(v, \mathbf{p}_0, \mathbf{p}_2) \qquad \mathbf{r}_1 = Lerp(v, \mathbf{p}_1, \mathbf{p}_3)$$



Bilinear patch

- Consider that $\mathbf{r}_0$, $\mathbf{r}_1$ define a line segment
- Evaluate it using *u* to get **x**

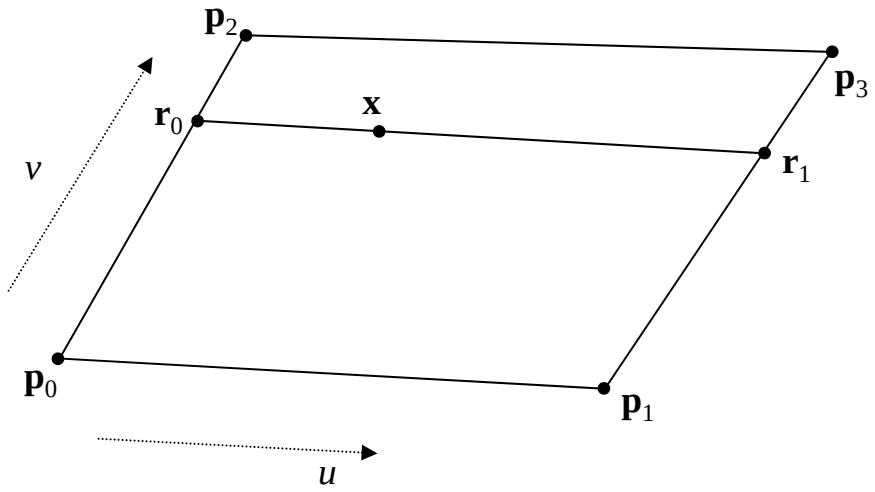
$$\mathbf{x} = Lerp(u, \mathbf{r}_0, \mathbf{r}_1)$$



Bilinear patch

- Formula for the *v* direction first

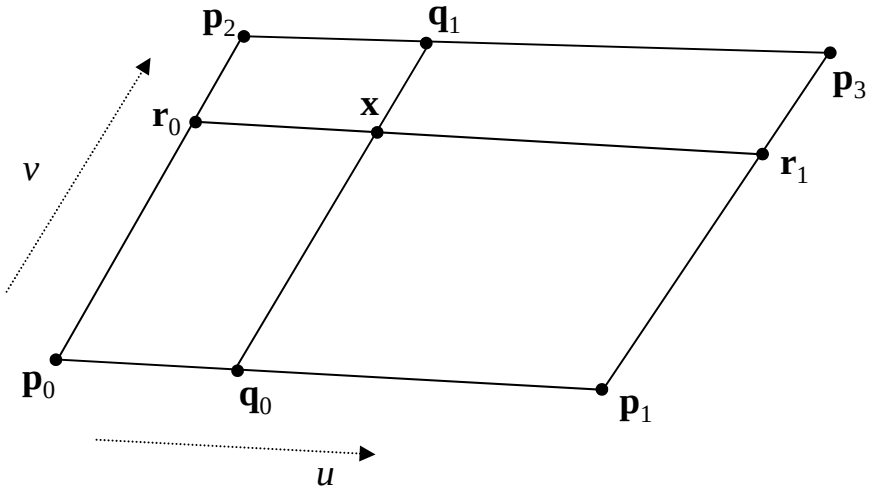
$$\mathbf{x}(u,v) = Lerp(u, Lerp(v, \mathbf{p}_0, \mathbf{p}_2), Lerp(v, \mathbf{p}_1, \mathbf{p}_3))$$



Bilinear patch

- Patch geometry is independent of the order of *u* and *v*

$$\mathbf{x}(u,v) = Lerp(v, Lerp(u, \mathbf{p}_0, \mathbf{p}_1), Lerp(u, \mathbf{p}_2, \mathbf{p}_3))$$
$$\mathbf{x}(u,v) = Lerp(u, Lerp(v, \mathbf{p}_0, \mathbf{p}_2), Lerp(v, \mathbf{p}_1, \mathbf{p}_3))$$



Bilinear patch

- Weighted sum of control points

$$\mathbf{x}(u,v) = (1-u)(1-v)\mathbf{p}_0 + u(1-v)\mathbf{p}_1 + (1-u)v\mathbf{p}_2 + uv\mathbf{p}_3$$

- Bilinear polynomial

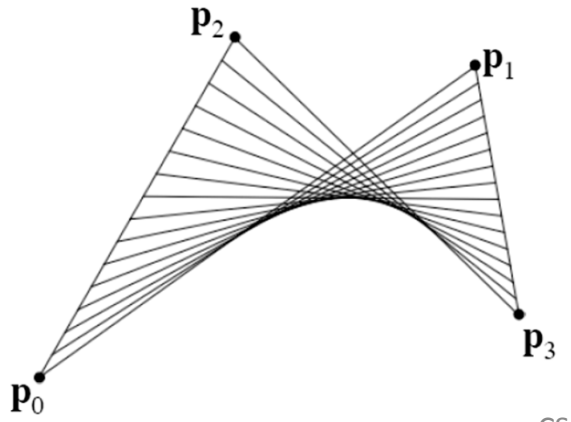
$$\mathbf{x}(u,v) = (\mathbf{p}_0 - \mathbf{p}_1 - \mathbf{p}_2 + \mathbf{p}_3)uv + (\mathbf{p}_1 - \mathbf{p}_0)u + (\mathbf{p}_2 - \mathbf{p}_0)v + \mathbf{p}_0$$

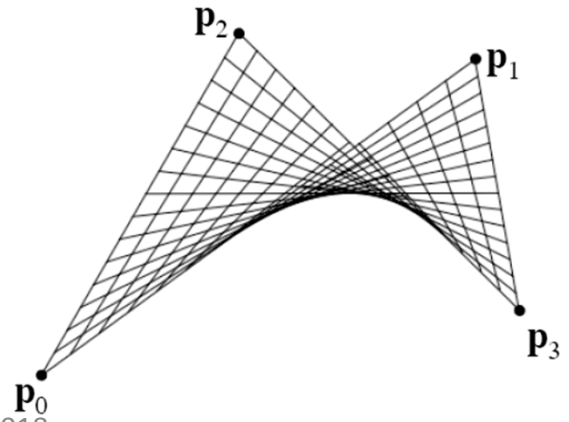
- Matrix form

$$x(u,v) = \begin{bmatrix} 1-u & u \end{bmatrix} \begin{bmatrix} p_0 & p_2 \\ p_1 & p_3 \end{bmatrix} \begin{bmatrix} 1-v \\ v \end{bmatrix}$$

Bilinear patch

- Properties
  - Patch interpolates the control points
  - The boundaries are straight line segments
  - If all 4 points of the control mesh are co-planar, the patch is flat
  - If the points are not co-planar, we get a curved surface
    - Saddle shape (hyperbolic paraboloid)
  - The parametric curves are all straight line segments
    - A (doubly) ruled surface: has (two) straight lines through every point
  - Not terribly useful as a modeling primitive



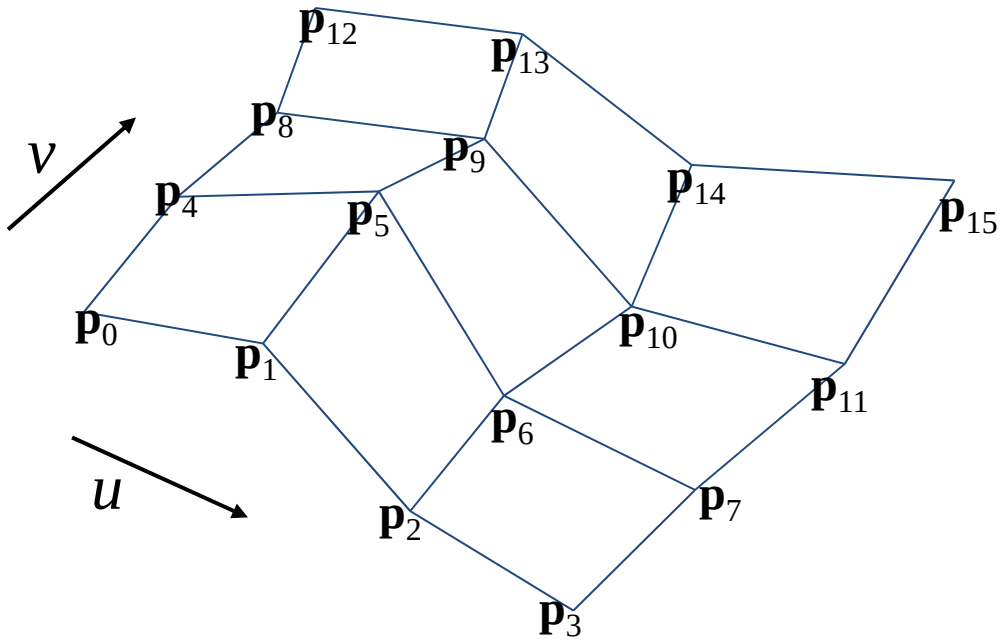


Bicubic Bézier patch

- Grid of 4x4 control points, $\mathbf{p}_0$ through $\mathbf{p}_{15}$
- Four rows of control points define Bézier curves along *u*

  $\mathbf{p}_0,\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3$; $\mathbf{p}_4,\mathbf{p}_5,\mathbf{p}_6,\mathbf{p}_7$; $\mathbf{p}_8,\mathbf{p}_9,\mathbf{p}_{10},\mathbf{p}_{11}$; $\mathbf{p}_{12},\mathbf{p}_{13},\mathbf{p}_{14},\mathbf{p}_{15}$
- Four columns define Bézier curves along *v*

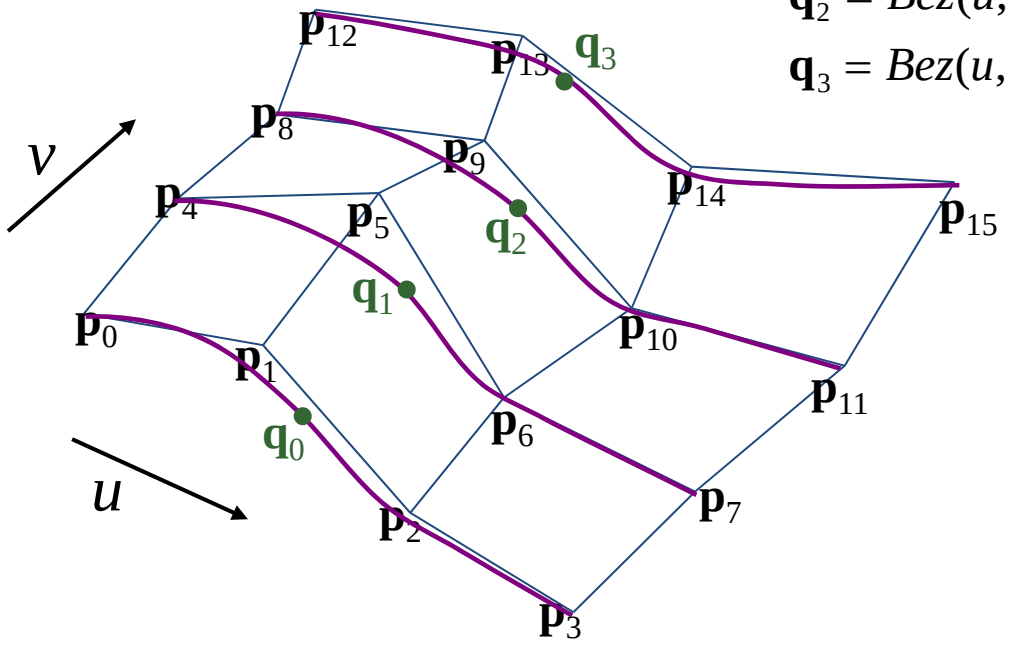
  $\mathbf{p}_0,\mathbf{p}_4,\mathbf{p}_8,\mathbf{p}_{12}$; $\mathbf{p}_1,\mathbf{p}_6,\mathbf{p}_9,\mathbf{p}_{13}$; $\mathbf{p}_2,\mathbf{p}_6,\mathbf{p}_{10},\mathbf{p}_{14}$; $\mathbf{p}_3,\mathbf{p}_7,\mathbf{p}_{11},\mathbf{p}_{15}$



Bicubic Bézier patch (step 1)

- Evaluate four *u*-direction Bézier curves at scalar value *u* [0..1]
- Get points $\mathbf{q}_0,\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3$

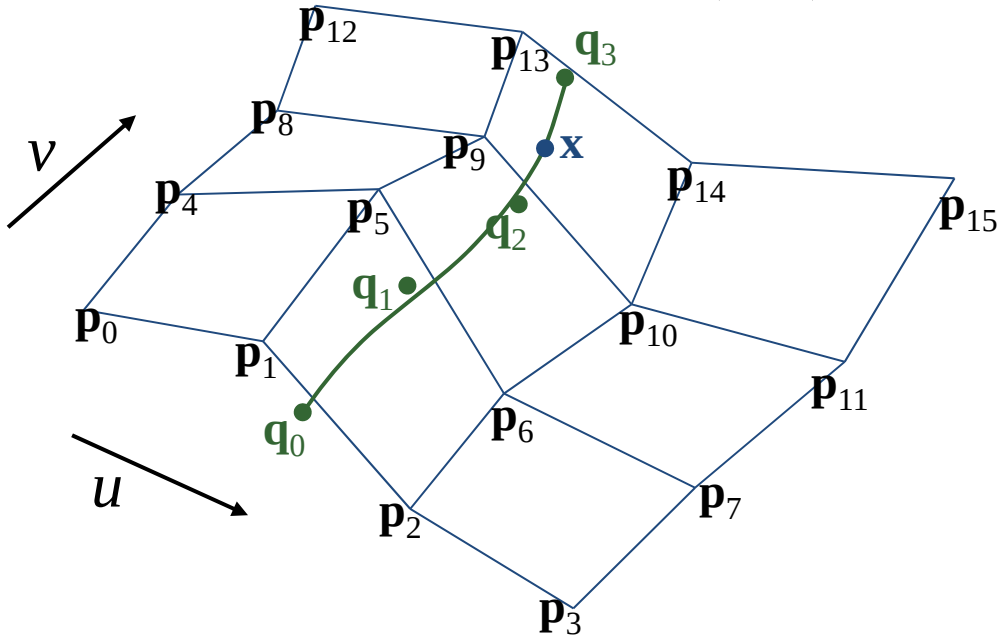
$$\mathbf{q_0} = Bez(u,\mathbf{p}_0,\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)$$
$$\mathbf{q}_1 = Bez(u,\mathbf{p}_4,\mathbf{p}_5,\mathbf{p}_6,\mathbf{p}_7)$$
$$\mathbf{q}_2 = Bez(u,\mathbf{p}_8,\mathbf{p}_9,\mathbf{p}_{10},\mathbf{p}_{11})$$
$$\mathbf{q}_3 = Bez(u,\mathbf{p}_{12},\mathbf{p}_{13},\mathbf{p}_{14},\mathbf{p}_{15})$$



Bicubic Bézier patch (step 2)

- Points $\mathbf{q}_0,\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3$ define a Bézier curve
- Evaluate it at v [0..1]

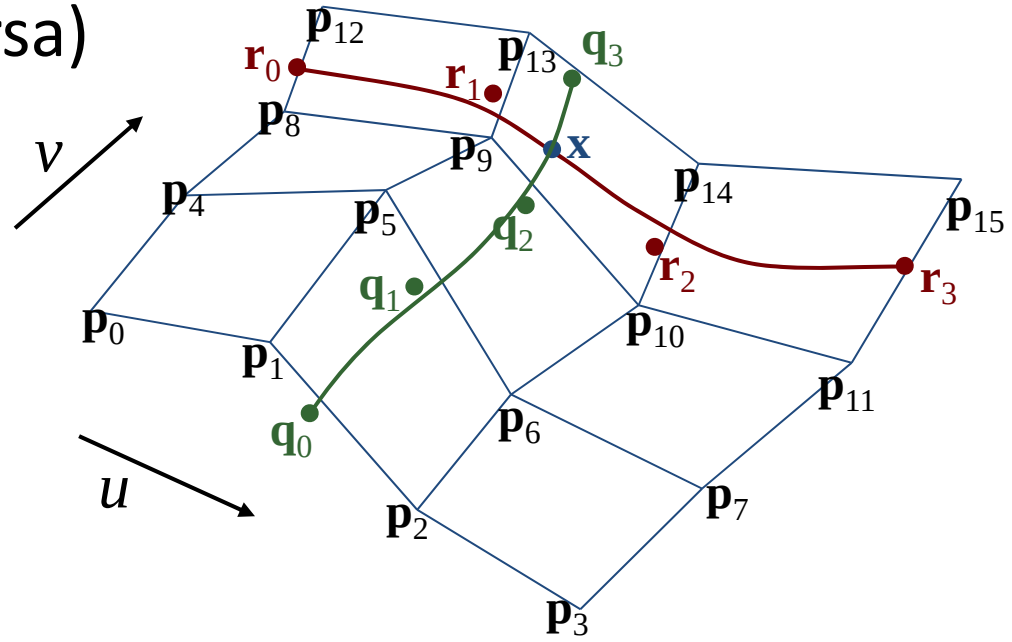
$$\mathbf{x}(u,v) = Bez(v, \mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$$



Bicubic Bézier patch

- Same result in either order (evaluate *u* before *v* or vice versa)

$$\begin{aligned}
\mathbf{q_0} &= Bez(u, \mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) & \mathbf{r_0} &= Bez(v, \mathbf{p}_0, \mathbf{p}_4, \mathbf{p}_8, \mathbf{p}_{12}) \\
\mathbf{q}_1 &= Bez(u, \mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6, \mathbf{p}_7) & \mathbf{r}_1 &= Bez(v, \mathbf{p}_1, \mathbf{p}_5, \mathbf{p}_9, \mathbf{p}_{13}) \\
\mathbf{q}_2 &= Bez(u, \mathbf{p}_8, \mathbf{p}_9, \mathbf{p}_{10}, \mathbf{p}_{11}) \quad \Leftrightarrow & \mathbf{r}_2 &= Bez(v, \mathbf{p}_2, \mathbf{p}_6, \mathbf{p}_{10}, \mathbf{p}_{14}) \\
\mathbf{q}_3 &= Bez(u, \mathbf{p}_{12}, \mathbf{p}_{13}, \mathbf{p}_{14}, \mathbf{p}_{15}) & \mathbf{r}_3 &= Bez(v, \mathbf{p}_3, \mathbf{p}_7, \mathbf{p}_{11}, \mathbf{p}_{15}) \\
\mathbf{x}(u,v) &= Bez(v, \mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) & \mathbf{x}(u,v) &= Bez(u, \mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)
\end{aligned}$$



Bicubic Bézier patch, matrix form

$$\mathbf{U} = \begin{bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} v^3 \\ v^2 \\ v \\ 1 \end{bmatrix} \quad \mathbf{B}_{Bez} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \mathbf{B}_{Bez}^T$$

$$\mathbf{C}_x = \mathbf{B}_{Bez}^T \mathbf{G}_x \mathbf{B}_{Bez}$$
$$\mathbf{C}_y = \mathbf{B}_{Bez}^T \mathbf{G}_y \mathbf{B}_{Bez}$$
$$\mathbf{C}_z = \mathbf{B}_{Bez}^T \mathbf{G}_z \mathbf{B}_{Bez}$$

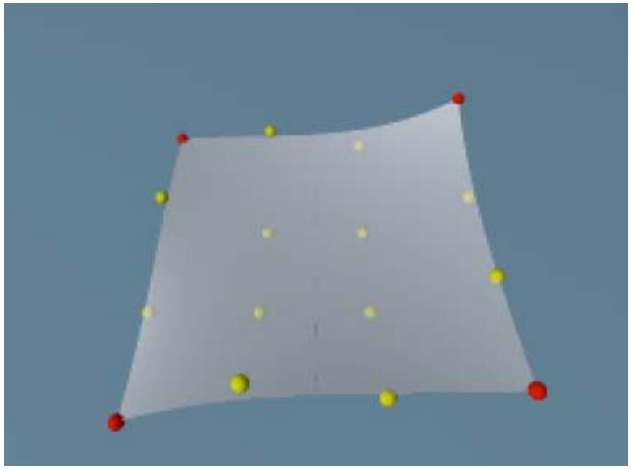
$$\mathbf{G}_x = \begin{bmatrix} p_{0x} & p_{1x} & p_{2x} & p_{3x} \\ p_{4x} & p_{5x} & p_{6x} & p_{7x} \\ p_{8x} & p_{9x} & p_{10x} & p_{11x} \\ p_{12x} & p_{13x} & p_{14x} & p_{15x} \end{bmatrix}, \quad \mathbf{G}_y = \text{L} \quad , \quad \mathbf{G}_z = \text{L}$$

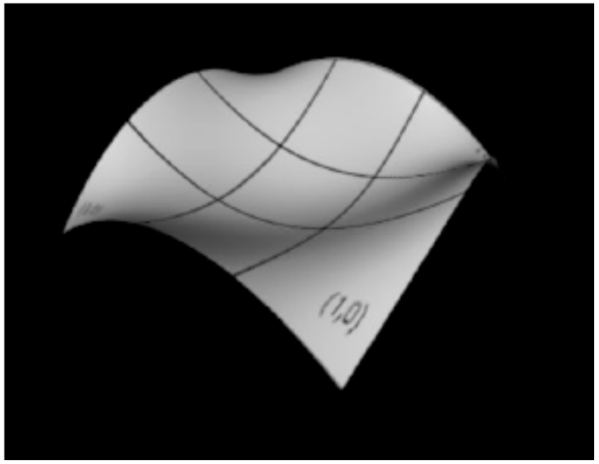
$$\mathbf{x}(u,v) = \begin{bmatrix} \mathbf{V}^T \mathbf{C}_x \mathbf{U} \\ \mathbf{V}^T \mathbf{C}_y \mathbf{U} \\ \mathbf{V}^T \mathbf{C}_z \mathbf{U} \end{bmatrix}$$

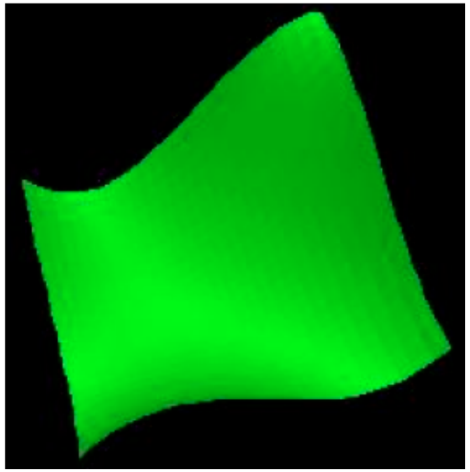
C stores the coefficients of the bicubic equation
G stores the control point geometry
$\mathbf{B}_{Bez}$ is the basis matrix (Bézier basis)
U and **V** are the vectors formed from the powers of u and v

Bicubic Bézier patch

- Properties
  - Convex hull
    - Any point on the surface will fall within the convex hull of the control points
  - Interpolates 4 corner points
  - Approximates other 12 points, which act as “handles”
  - The boundaries of the patch are the Bézier curves defined by the points on the mesh edges
  - The parametric curves are all Bézier curves

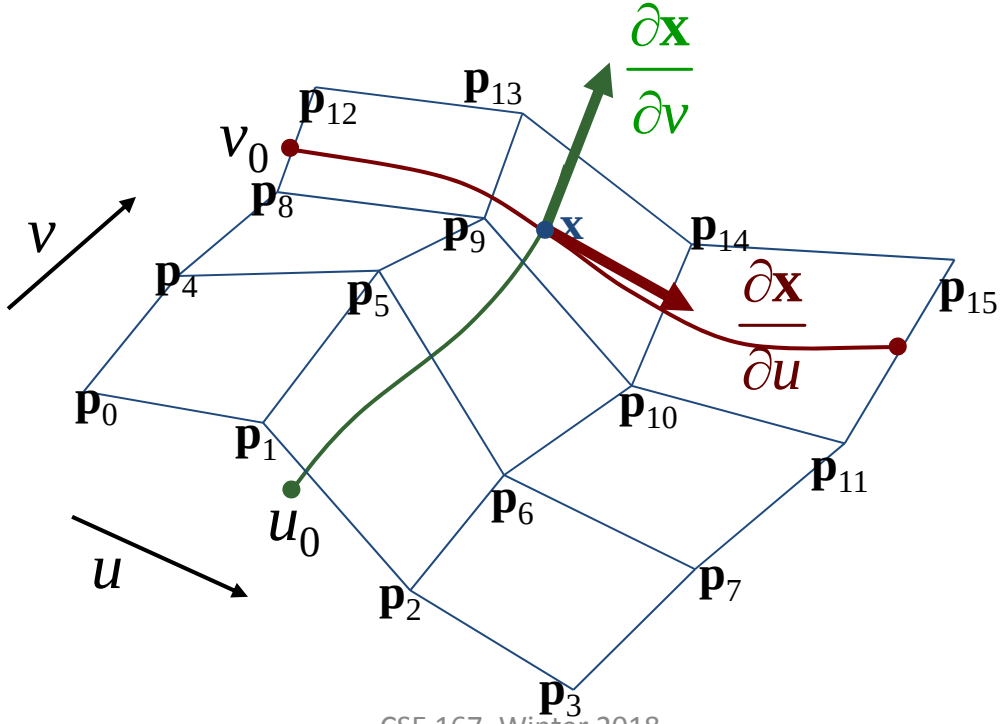






Tangents of a Bézier patch

- Recall parametric curves $\mathbf{x}(u,v_0)$ and $\mathbf{x}(u_0,v)$, where v_0 and u_0 are fixed, respectively
- Tangents to surface = tangents to parametric curves
- Tangents are partial derivatives of $\mathbf{x}(u,v)$
- Normal is cross product of the tangents



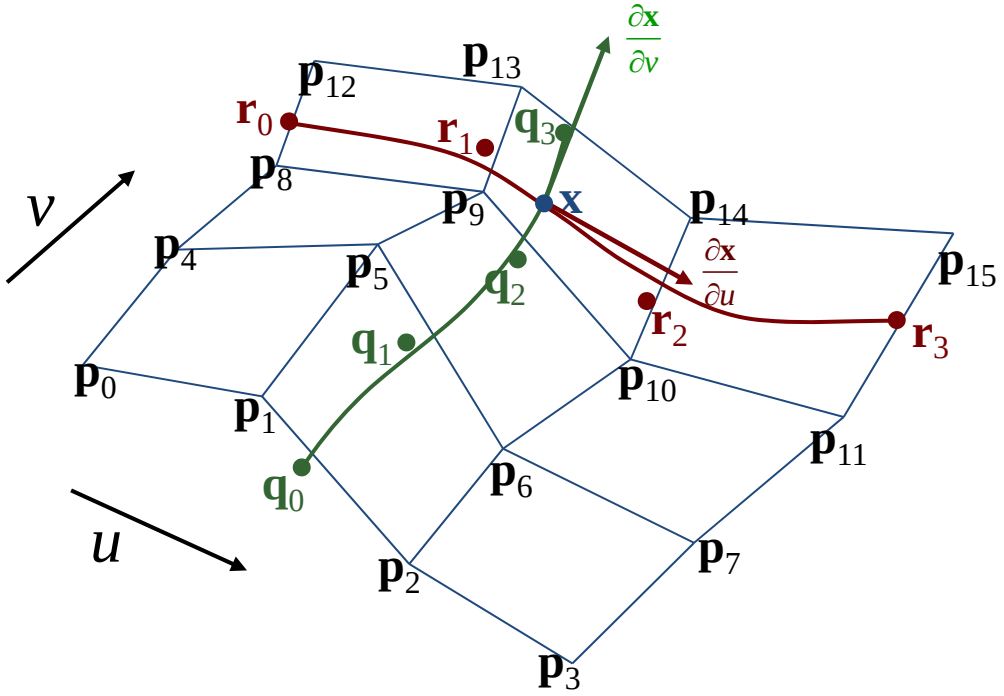
Tangents of a Bézier patch

$$\mathbf{q_0} = Bez(u, \mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$$
$$\mathbf{q}_1 = Bez(u, \mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6, \mathbf{p}_7)$$
$$\mathbf{q}_2 = Bez(u, \mathbf{p}_8, \mathbf{p}_9, \mathbf{p}_{10}, \mathbf{p}_{11})$$
$$\mathbf{q}_3 = Bez(u, \mathbf{p}_{12}, \mathbf{p}_{13}, \mathbf{p}_{14}, \mathbf{p}_{15})$$

$$\mathbf{r_0} = Bez(v, \mathbf{p}_0, \mathbf{p}_4, \mathbf{p}_8, \mathbf{p}_{12})$$
$$\mathbf{r}_1 = Bez(v, \mathbf{p}_1, \mathbf{p}_5, \mathbf{p}_9, \mathbf{p}_{13})$$
$$\mathbf{r}_2 = Bez(v, \mathbf{p}_2, \mathbf{p}_6, \mathbf{p}_{10}, \mathbf{p}_{14})$$
$$\mathbf{r}_3 = Bez(v, \mathbf{p}_3, \mathbf{p}_7, \mathbf{p}_{11}, \mathbf{p}_{15})$$

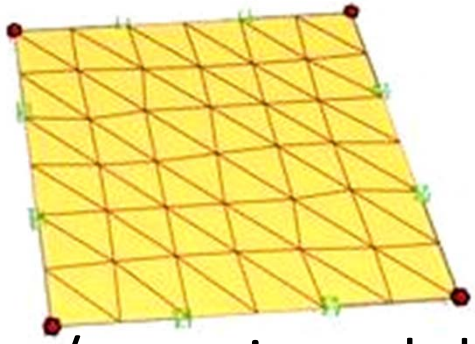
$$\frac{\partial \mathbf{x}}{\partial v}(u,v) = Bez'(v, \mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$$

$$\frac{\partial \mathbf{x}}{\partial u}(u,v) = Bez'(u, \mathbf{r}_0, \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$



Tessellating a Bézier patch

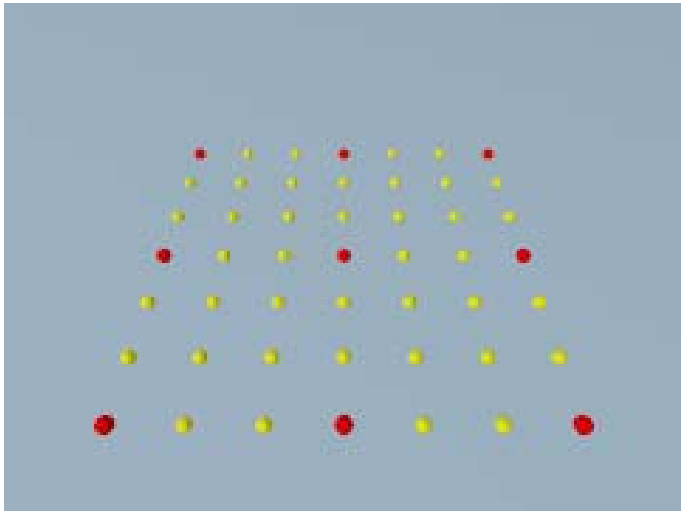
- Uniform tessellation is most straightforward
  - Evaluate points on a grid of u,v coordinates
  - Compute tangents at each point, take cross product to get per-vertex normal
  - Draw triangle strips with glBegin(GL_TRIANGLE_STRIP)



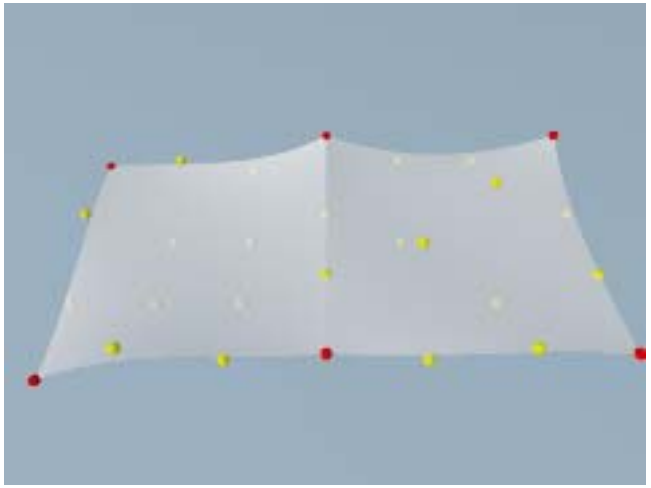
- Adaptive tessellation/recursive subdivision
  - Potential for “cracks” if patches on opposite sides of an edge divide differently
  - Tricky to get right, but can be done

Piecewise Bézier surface

- Lay out grid of adjacent meshes of control points
- For C_0 continuity, must share points on the edge
  - Each edge of a Bézier patch is a Bézier curve based only on the edge mesh points
  - So if adjacent meshes share edge points, the patches will line up exactly
- But, results in a crease



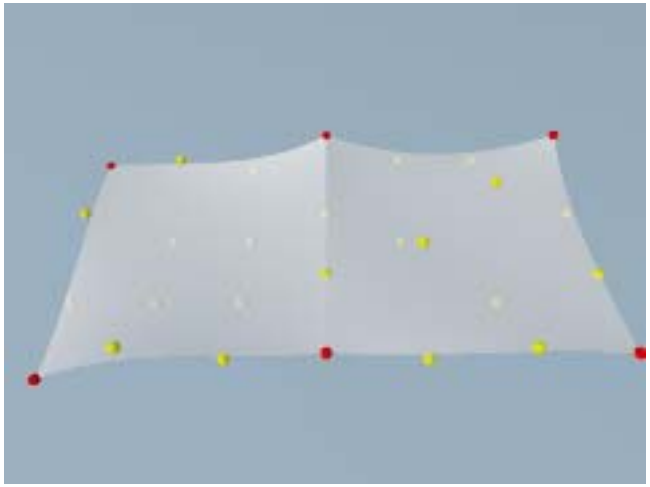
Grid of control points

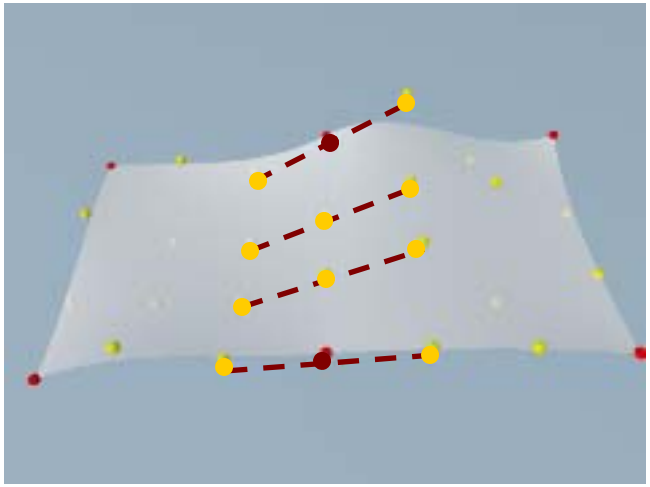


Piecewise Bézier surface

C_1 continuity

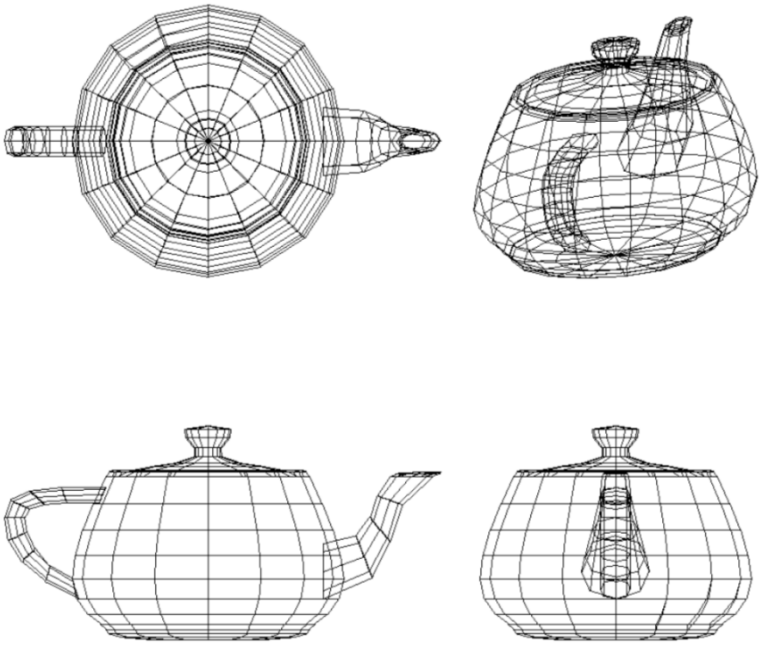
- We want the parametric curves that cross each edge to have C_1 continuity
  - So the handles must be equal-and-opposite across the edge





Modeling with Bézier patches

- Original Utah teapot, from Martin Newell's PhD thesis, consisted of 28 Bézier patches.
- The original had no rim for the lid and no bottom
- Later, four more patches were added to create a bottom, bringing the total to 32
- The data set was used by a number of people, including graphics guru Jim Blinn. In a demonstration of a system of his he scaled the teapot by .75, creating a stubbier teapot. He found it more pleasing to the eye, and it was this scaled version that became the highly popular dataset used today.



Pixar's walking teapot

Source: http://www.holmes3d.net/graphics/teapot/