CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/

Today's learning goals

Sipser Section 1.4

- Apply the Pumping Lemma in proofs of nonregularity
- Identify some nonregular sets

Pumping Lemma

Sipser p. 78 Theorem 1.70

If A is a regular language, then there is a number p (*the pumping length*) where, if s is any string in A of length at least p, then s may be divided into three pieces, $\boxed{s = x\ y\ z}$ such that

- $|y| > 0$, and
- for each $i \geq 0$, $xy^iz \in A$,
- $|xy| \leq p$.

Last week's example

Claim: The set L = **$\{0^n1^n \mid n \geq 0\}$** is not regular.

Proof: Consider an arbitrary positive integer. WTS p is not a pumping length for L. Consider the string $s = 0^p1^p$. Then, s is in L and |s| = 2p >= p. Consider any division of s into three parts

$$s = xyz \text{ with } |y|>0, |xy| \leq p.$$

Since $|xy| \leq p$, $x=0^k$, $y = 0^m$, $z = 0^r1^p$ with $k+m+r = p$,

and since $|y| > 0$, $m>0$. Picking i=0: $xy^iz = xz = 0^k0^r1^p = 0^{k+r}1^p$, which is not in L because $k+r < p$. Thus, no p can be a pumping length for L and L is not regular.

Proof strategy

To prove that a language L is **not** regular

- Consider arbitrary positive integer p.
- Prove that p isn't a pumping length for L.

- Conclude that L does not have any pumping length and is therefore not regular.

Another example

Claim: The set $\{a^n b^m a^n \mid m,n \geq 0\}$ is not regular.

Proof: Consider an arbitrary positive integer. WTS p is not a pumping length for L.

Consider the string

s = ???

1. |s| >= p ?
2. s is in L ?
3. No matter how we cut s into three (viable) pieces, some related string obtained by "repeating" the middle part falls out of L ?

Aside…

To complete proofs with Pumping Lemma, we will need to build (useful) examples of strings with **length >= p** that are **in** a given language.

- L1 = $\{a^n b^m a^n \mid m,n \geq 0\}$
- L2 = $\{ ww \mid w \text{ is a string over } \{0,1\} \}$
- L3 = $\{ ww^R \mid w \text{ is a string over } \{0,1\} \}$

Another example

Claim: The set $\{a^n b^m a^n \mid m,n \geq 0\}$ is not regular.

Proof: … You must pick s carefully: we want $|s| \geq p$ and s in L and s "can't be pumped"

Which choices of s can be used to complete the proof?

A. $s = a^p b^p$ B. $s = aba$ C. $s = a^p b^p a^p$ D. $s = b^p$

E. None of the above

And another

Claim: The set {w w^R | w is a string over {0,1} } is not regular.

Proof: … You must pick s carefully: we want |s|≥p and s in L and s "can't be pumped" … **Consider i=…**

Which s and i let us complete the proof?

A. s = 0^p0^p, i=2 B. s = 0110, i=0 C. s = 0^p110^p, i=1 D. s = 1^p001^p, i=3

E. I don't know

How do we choose i?

Claim: The set $\{0^j 1^k \mid j,k \geq 0$ and $j \geq k\}$ is not regular.

Proof: … You must pick s carefully: we want $|s| \geq p$ and s in L and s "can't be pumped" … **Consider i=…**

Which s and i let us complete the proof?

A. $s = 0^p 1^p$, i=2 B. $s = 0^p 1^p$, i=p C. $s = 0^p 1^p$, i=1 D. $s = 0^p 1^p$, i=0

E. I don't know

Do we always need Pumping Lemma?

Claim: The set

{w | w has different #s of 0s and 1s OR has a 1 before a 0}

is nonregular.

Proof:

Regular sets: not the end of the story

- Many **nice / simple / important** sets are not regular
- Limitation of the finite-state automaton model
  - Can't "count"
  - Can only remember finitely far into the past
  - Can't backtrack
  - Must make decisions in "real-time"
- We know computers are more powerful than this model…

Which conditions should we relax?

For next time

- Work on Individual HW3 **due Tuesday**

Pre class-reading for Wednesday: pages 111-112.