CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/

Today's learning goals

Sipser Section ~~1.1~~ 1.2

- Design NFA recognizing a given language
- Convert an NFA (with or without spontaneous moves) to a DFA recognizing the same language

NFA

Formal definition?

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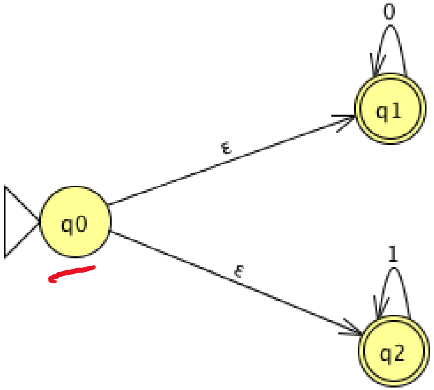

only 0s

only 1s

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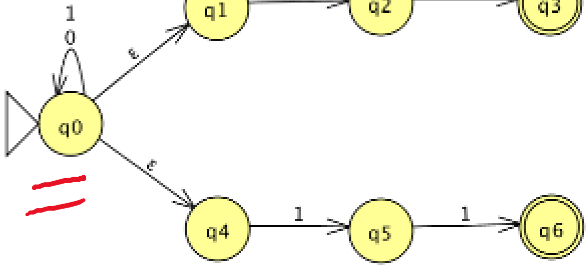

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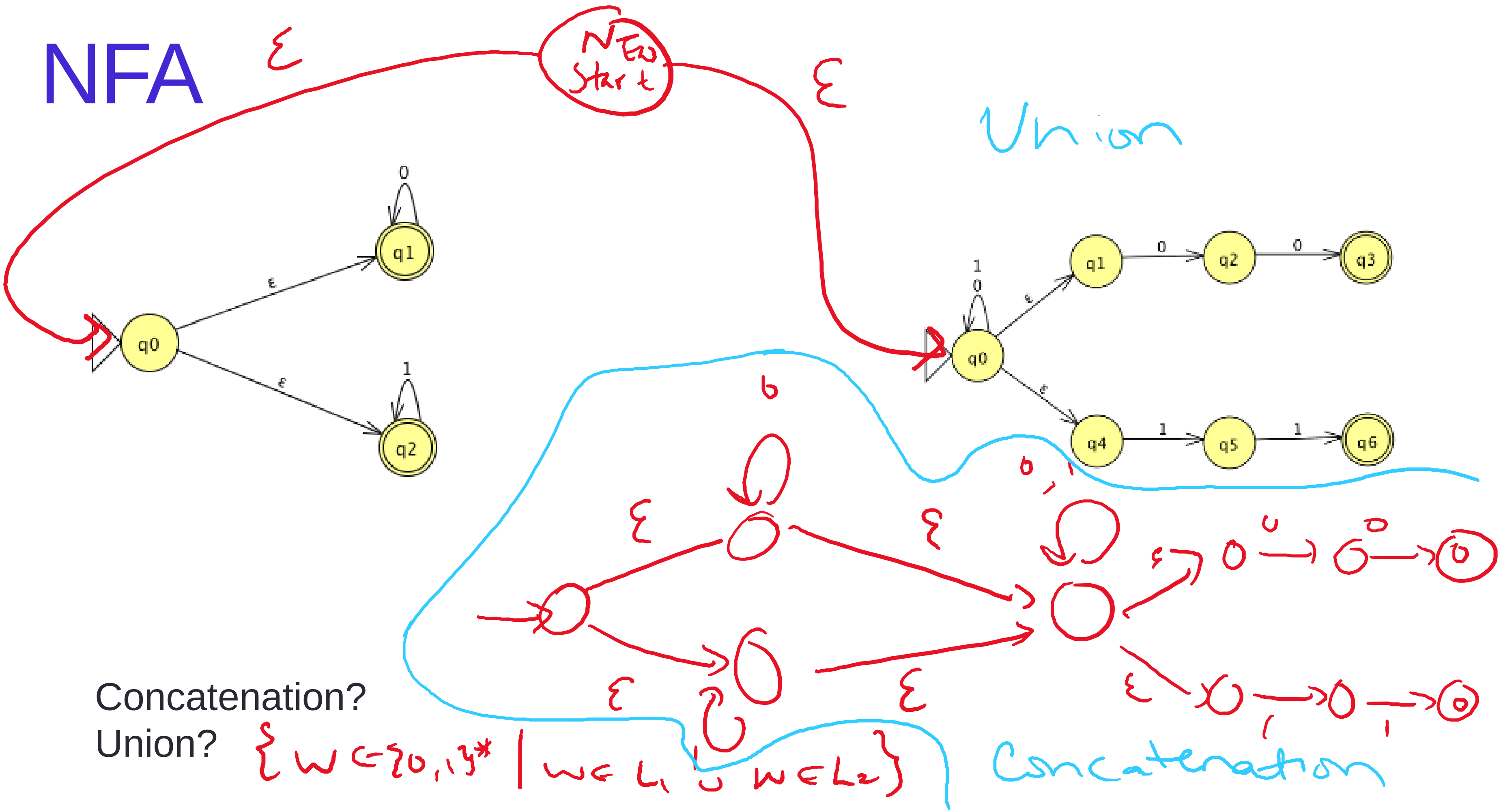
"Set builder notation"

L(M1) = $\{w \in \{0,1\}^* \mid w = 0^n$ or $w = 1^n$ for $n \geq 0\}$

L(M2) = $\{w \in \{0,1\}^* \mid$ last two chars of w are equal$\}$

NFA



Concatenation?

Union? $\{w \in \{0,1\}^* \mid w \in L_1 \cup w \in L_2\}$

Simulating NFA with DFA

Not quite a closure proof, but …

Proof:

Given name variables for sets, machines assumed to exist.

WTS state goal and outline plan.

Construction using objects previously defined + new tools working towards goal. Give formal definition and explain.

Correctness prove that construction works.

Conclusion recap what you've proved.

Simulating NFA with DFA

For any language recognized by an NFA, there is a DFA that recognizes this language.

Proof:

Given A, a language recognized by $N = (Q,\Sigma,\delta,q0,F)$ a NFA

WTS there is some DFA M with $L(M) = A$

Construction

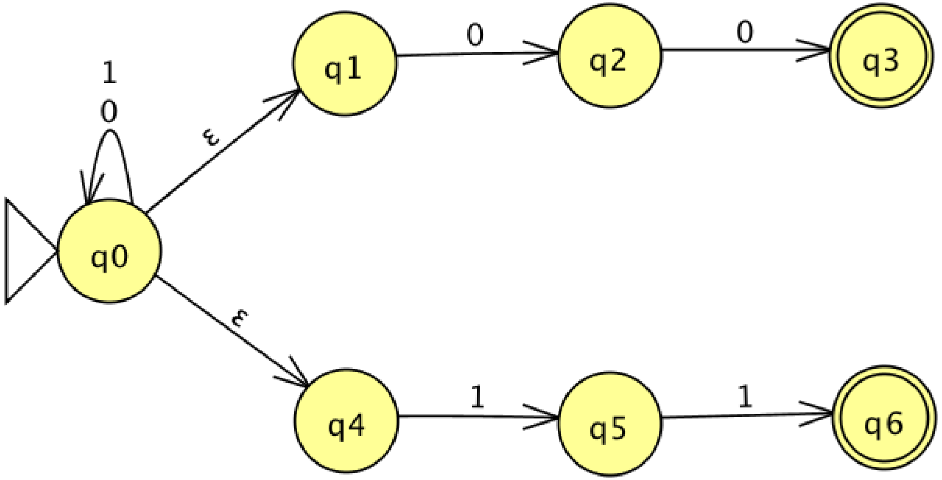
Correctness

Conclusion

Idea of construction

Track set of **possible** states NFA might be in.

From NFA to DFA



Which states can this NFA be in before first input symbol is read?

A. q0
B. any state
C. q0, q1
D. q0, q4
E. q0, q1, q4

Subset construction

Given A, a language recognized by N = (Q,Σ,δ,q0,F) a NFA

WTS there is some DFA M with L(M) = A

Construction Define $\boxed{M}$ = (Q', Σ, δ',q0', F') with

- $\boxed{Q'}$ = the power set of Q = { X | X is a subset of Q }
- q0' = { states N can be in before first input symbol read }
- F' = { }
- δ' ( ) =

Subset construction

Given A, a language recognized by $N = (Q,\Sigma,\delta,q0,F)$ a NFA

WTS there is some DFA M with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta',q0', F')$ with

- Q' = the power set of Q = { X | X is a subset of Q }
- $q0' = \{ q0 \} \cup \delta((q0, \varepsilon))$ …
- $F' = \{ \quad \}$
- $\delta' (\quad) =$

ex: $\{q_0, q_1, q_4\}$

Subset construction

Given A, a language recognized by $N = (Q,\Sigma,\delta,q0,F)$ a NFA

WTS there is some DFA M with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta',q0', F')$ with

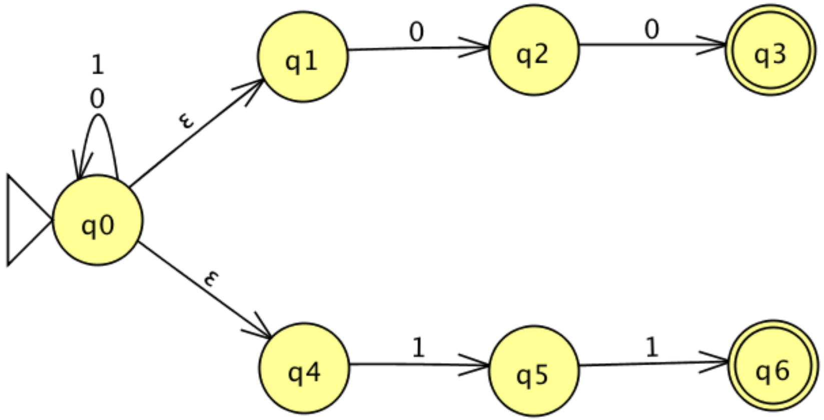
- Q' = the power set of Q = { X | X is a subset of Q }
- $q0' = \{ q0 \} \cup \delta((q0, \varepsilon))$ …
- $F' = \{ \dots \}$

original NFA's states   original transition function

- $\delta'((X, x))$ = { q in Q | q is in $\delta((r,x))$ for some r in X or is accessible via spontaneous moves}

Types? $X \in \mathcal{P}(Q)$ $\boxed{x} \in \Sigma$

From NFA to DFA



$\{q_0, q_1, q_4\}$

0 1

$\{q_0, q_1, q_2, q_4\}$ $\{q_0, q_1, q_4, q_5\}$

ex.

Subset construction

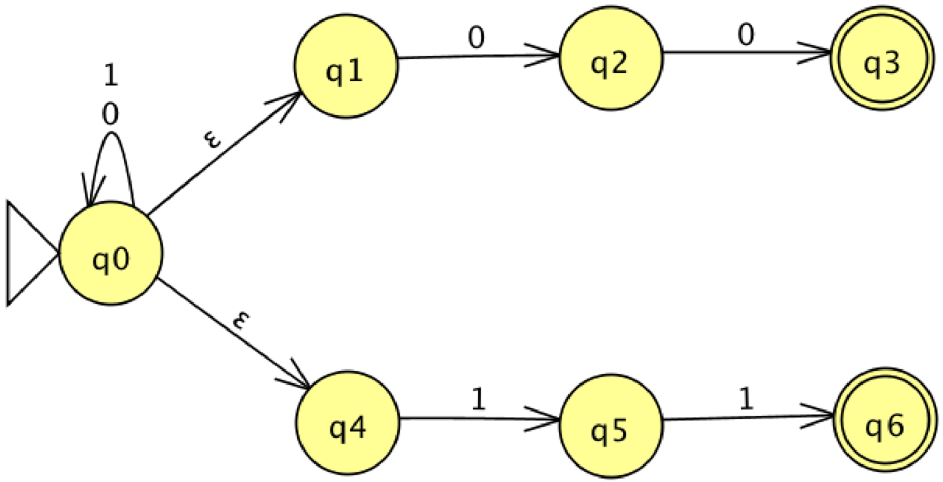
Given A, a language recognized by N = (Q,Σ,δ,q0,F) a NFA

WTS there is some DFA M with L(M) = A

Construction Define M = (Q', Σ, δ',q0', F') with

- Q' = the power set of Q = { X | X is a subset of Q }
- q0' = { q0 } U δ((q0, ε)) …
- F' = { guarantee at least one computation is successful }
- δ' ((X, x)) = { q in Q | q is in δ((r,x)) for some r in X or
  is accessible via spontaneous moves}

From NFA to DFA



current state in DFA

What does it mean for a set of states X to guarantee at least one computation is successful?

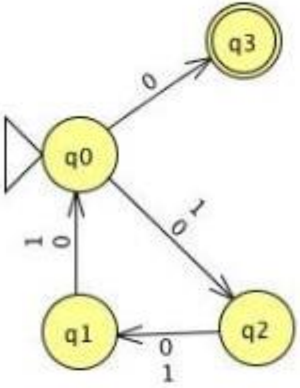
set of accept states in NFA

A. X is a subset of F
B. X = F
C. X ∩ F is nonempty
D. X is an element of F
E. None of the above.

i.e. $\{X \in P(Q) \mid X \cap F \neq \emptyset\}$

e.g. is $\{q_0, q_1, q_4\}$ in F'? No

Subset construction examples



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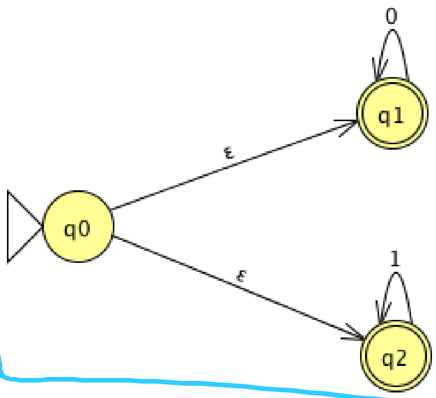

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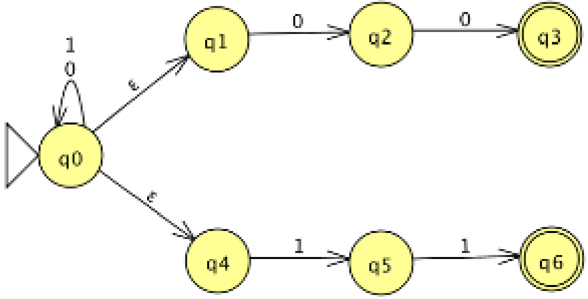

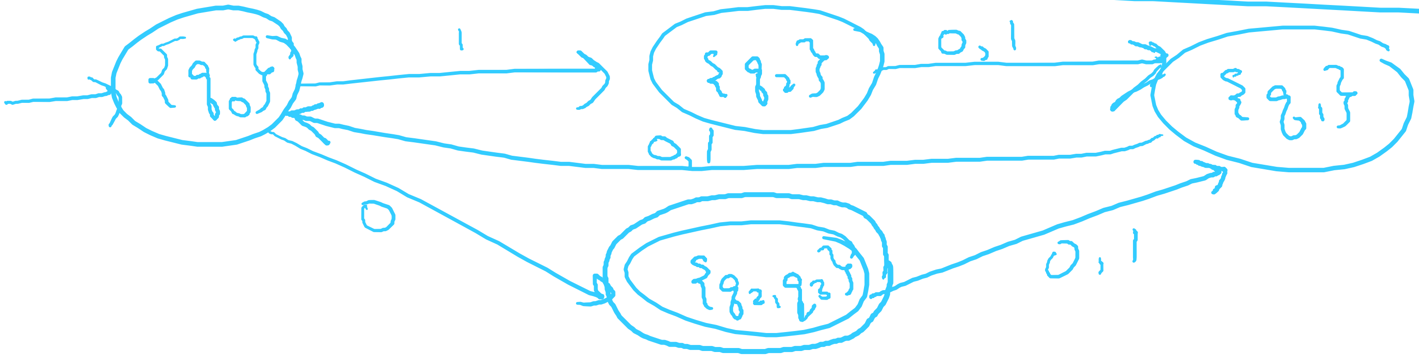

DFA equiv NFA

Theorem: For each language L,

L is recognizable by some DFA

iff

L is recognizable by some NFA

DFA equiv NFA equiv RegExp

Theorem: For each language L,

L is recognizable by some DFA

iff

L is recognizable by some NFA

iff

L is describable by some regular expression

For next time

- Work on Individual HW2 **due Tuesday**

Pre class-reading for Wednesday: Example 1.56 on page 68