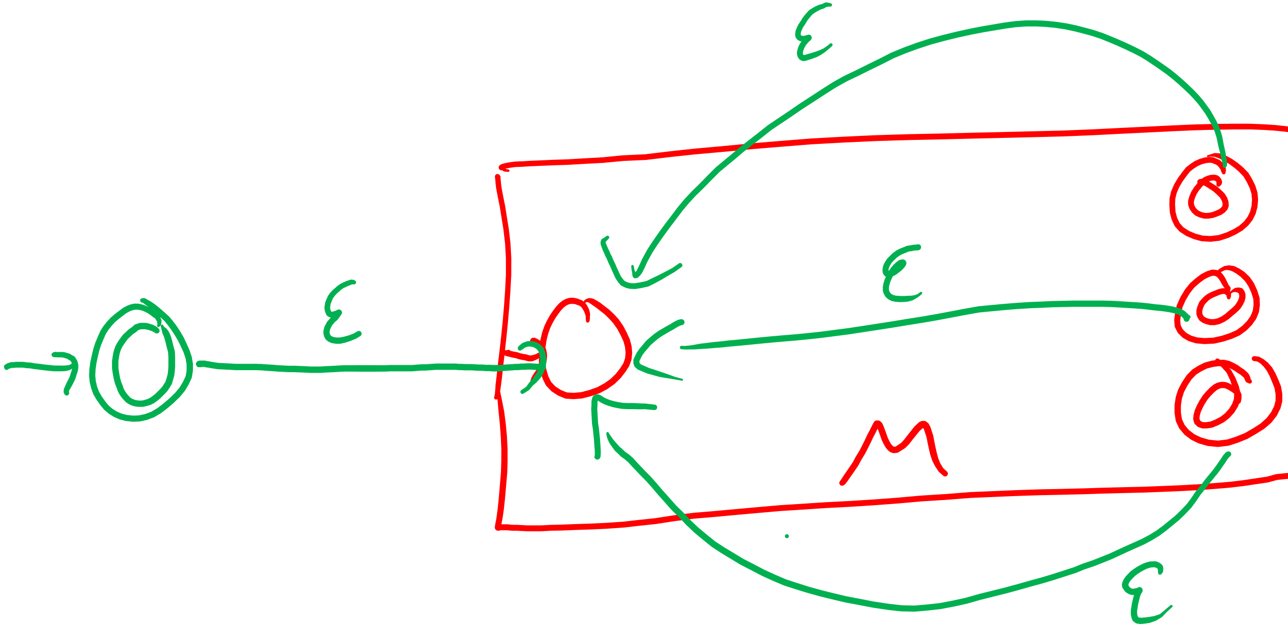
CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/

M recognizes L

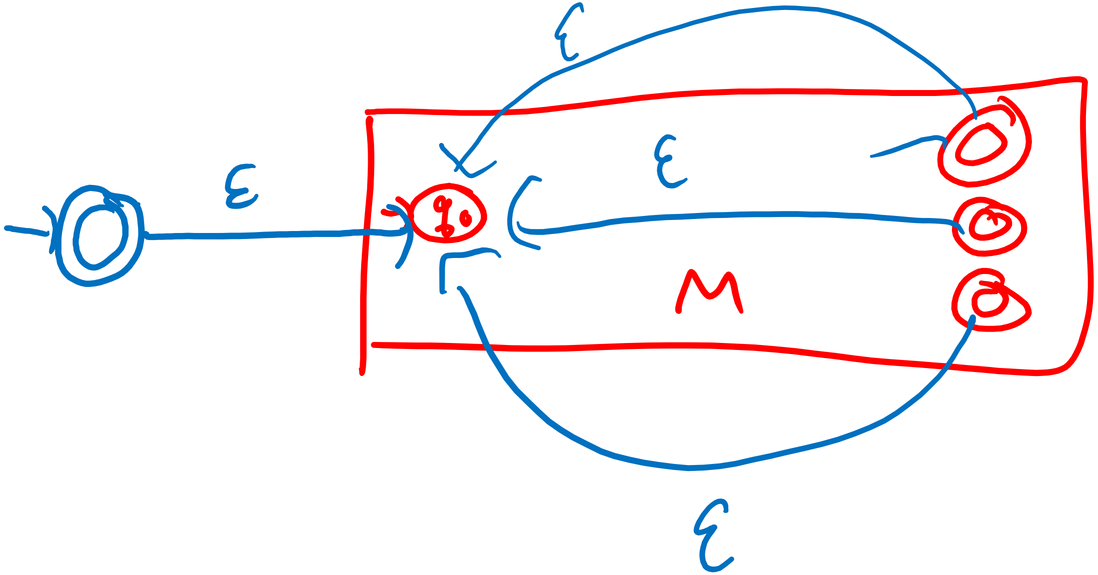
construct a machine that recognizes L^*



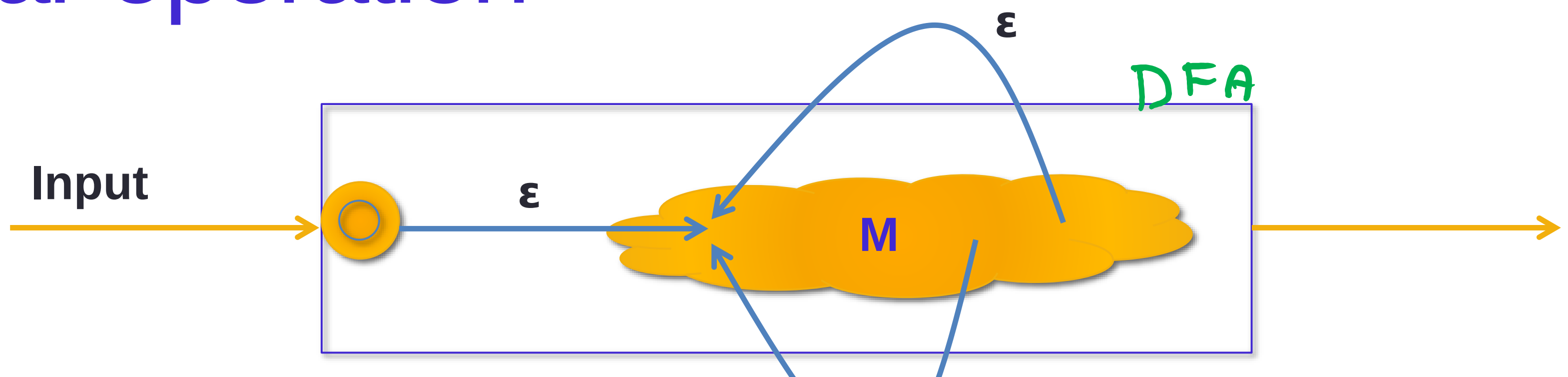
Star operation

M recognizes L

construct N that recognizes L^*



Star operation



Given $M1 = (Q1,\Sigma,\delta 1,\boxed{q1},F1)$, build

$N = (Q1 \cup \{q0\}, \Sigma, \delta, q0, F1 \cup \{q0\})$

and $\delta(q,x) = \ldots$ Exercise

$$\begin{cases} \{\delta 1(q,x)\} & \text{if } x \neq \varepsilon \\ \{q1\} & \text{if } x=\varepsilon,\ q \in F \text{ or } q = q0 \\ \{\} & \text{if } x=\varepsilon \text{ and } q \notin F \text{ and } q \neq q0 \end{cases}$$

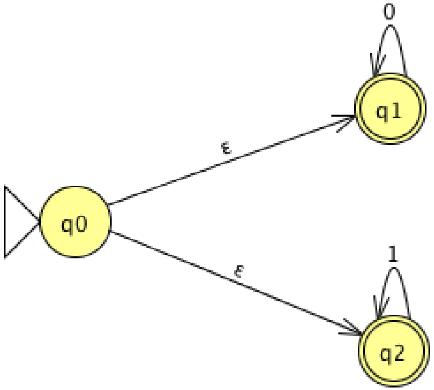
~~*Construction in the book (page 63)*~~

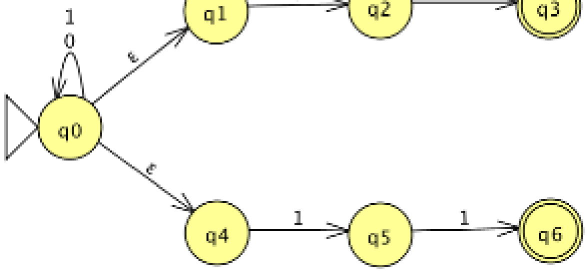
Today's learning goals

Sipser Section 1.1

- Design NFA recognizing a given language
- Convert an NFA (with or without spontaneous moves) to a DFA recognizing the same language

NFA



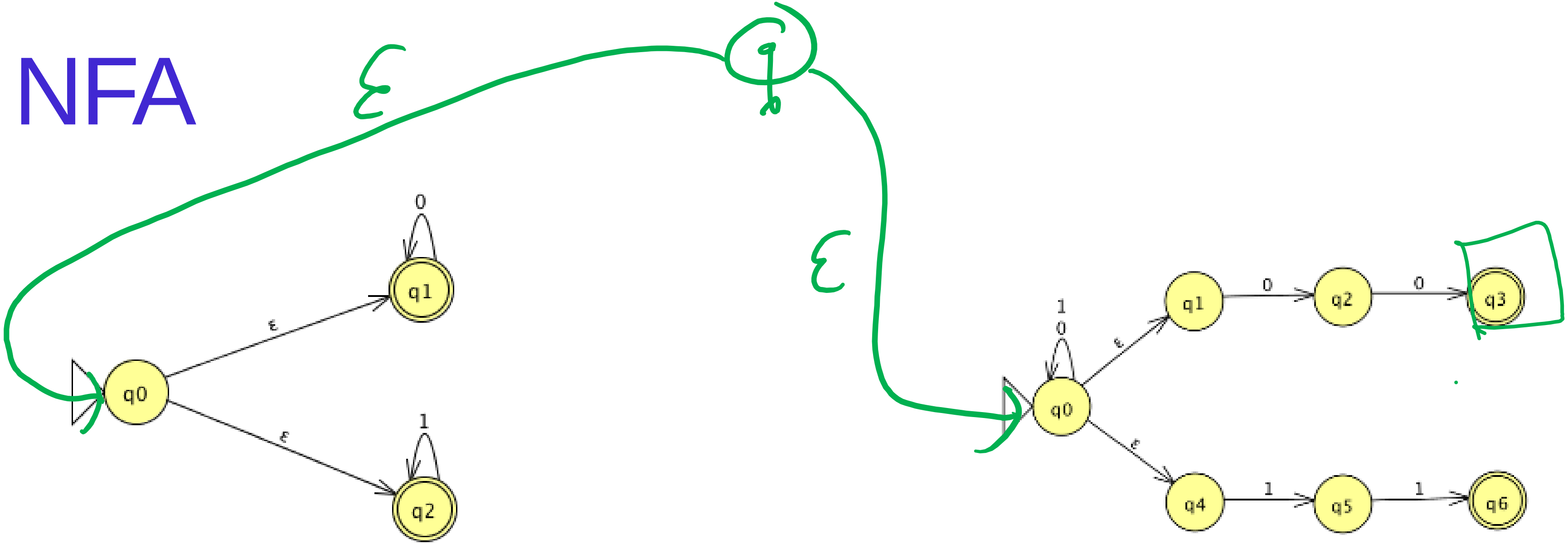


L(M1) = $\{ w \mid w = 0^n$ for some $n \in \mathbb{Z}$ or $w = 1^n$ for some $n \in \mathbb{Z} \}$

$0^* \cup 1^*$

L(M2) = $(0 \cup 1)^* (00 \cup 11)$

NFA



Union?

Simulating NFA with DFA

Not quite a closure proof, but …

Proof:

Given name variables for sets, machines assumed to exist.

WTS state goal and outline plan.

Construction using objects previously defined + new tools working towards goal. Give formal definition and explain.

Correctness prove that construction works.

Conclusion recap what you've proved.

Simulating NFA with DFA

For any language recognized by an NFA, there is a DFA that recognizes this language.

Proof:

Given A, a language recognized by $N = (Q,\Sigma,\delta,q0,F)$ a NFA

WTS there is some DFA M with L(M) = A

Construction

Correctness

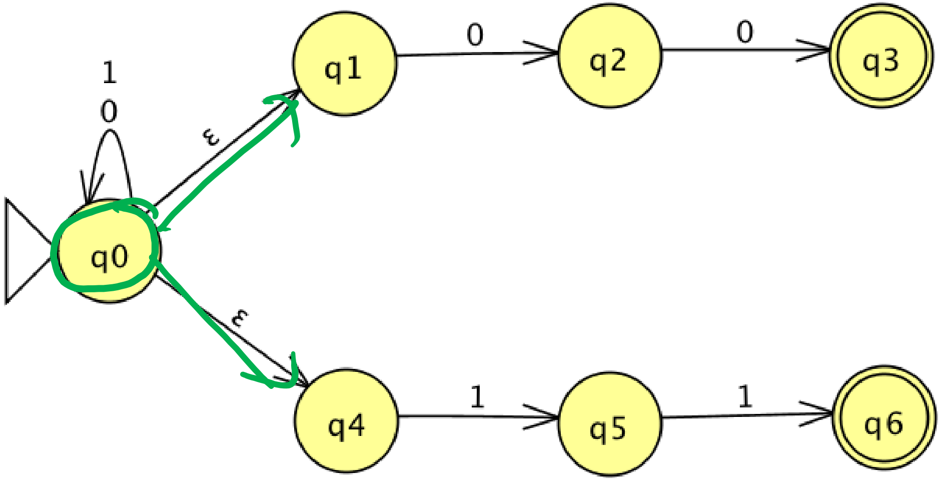
Conclusion

Idea of construction

Track set of **possible** states NFA might be in.

in our new machine (DFA) the set of states $\subseteq \mathcal{P}(Q)$

From NFA to DFA



DFA

→ {q0, q1, q4}

Which states can this NFA be in before first input symbol is read?

A. q0
B. any state
C. q0, q1
D. q0, q4
E. q0, q1, q4

Subset construction

Given A, a language recognized by N = (Q,Σ,δ,q0,F) a NFA

WTS there is some DFA M with L(M) = A

Construction Define M = (Q', Σ, δ',q0', F') with

- Q' = the power set of Q = { X | X is a subset of Q }
- q0' = { states N can be in before first input symbol read }
- F' = { }
- δ' ( ) =

→ {q0} ∪ {the set of all states that are ε-transitions from q0}

δ(q0, ε)

Subset construction

Given A, a language recognized by $N = (Q,\Sigma,\delta,q0,F)$ a NFA

WTS there is some DFA M with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta',q0', F')$ with

- Q' = the power set of $Q = \{ X \mid X \text{ is a subset of } Q \}$
- $q0' = \{ q0 \} \cup \delta((q0, \varepsilon))$ ...
- $F' = \{$ subsets that contain (at least one) original accept states. $\}$
- $\delta' (\qquad) =$

Subset construction

Given A, a language recognized by $N = (Q,\Sigma,\delta,q0,F)$ a NFA

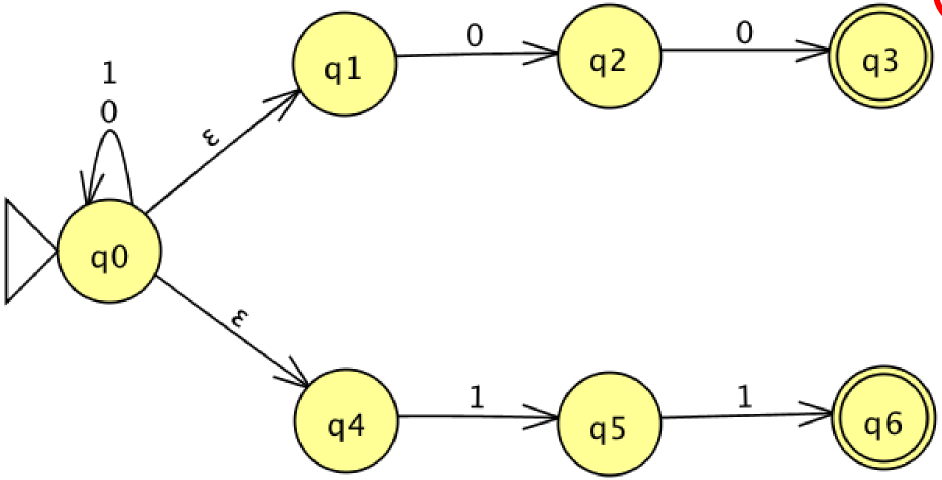
WTS there is some DFA M with $L(M) = A$

Construction Define $M = (Q', \Sigma, \delta', q0', F')$ with

- Q' = the power set of $Q = \{ X \mid X \text{ is a subset of } Q \}$
- $q0' = \{ q0 \} \cup \delta((q0, \varepsilon))$ …
- $F' = \{$ guarantee at least one computation is successful $\}$
- $\delta' (\qquad\qquad) =$

From NFA to DFA

X is a state in the new DFA



What does it mean for a subset of states X to guarantee at least one computation is successful?

A. X is a subset of F
B. X = F
C. X ∩ F is nonempty
D. X is an element of F
E. None of the above.

Subset construction

Given A, a language recognized by N = $(Q,\Sigma,\delta,q0,F)$ a NFA

WTS there is some DFA M with L(M) = A

Construction Define M = $(Q', \Sigma, \delta', q0', F')$ with

- Q' = the power set of Q = { X | X is a subset of Q }
- $q0' = \{ q0 \} \cup \delta((q0, \varepsilon))$ …
- F' = { X | X is a subset of Q and $X \cap F$ is nonempty }
- $\delta'(\,(X, x)\,) = \{ \quad \}$

Subset construction

Given A, a language recognized by N = (Q,Σ,δ,q0,F) a NFA

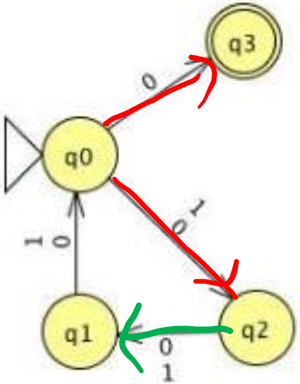
WTS there is some DFA M with L(M) = A

Construction Define M = (Q', Σ, δ',q0', F') with

- Q' = the power set of Q = { X | X is a subset of Q }
- q0' = { q0 } U δ((q0, ε)) …
- F' = { X | X is a subset of Q and X ∩ F is nonempty }
- δ' ((X, x)) = { q in Q | q is in δ((r,x)) for some r in X or is accessible via spontaneous moves}

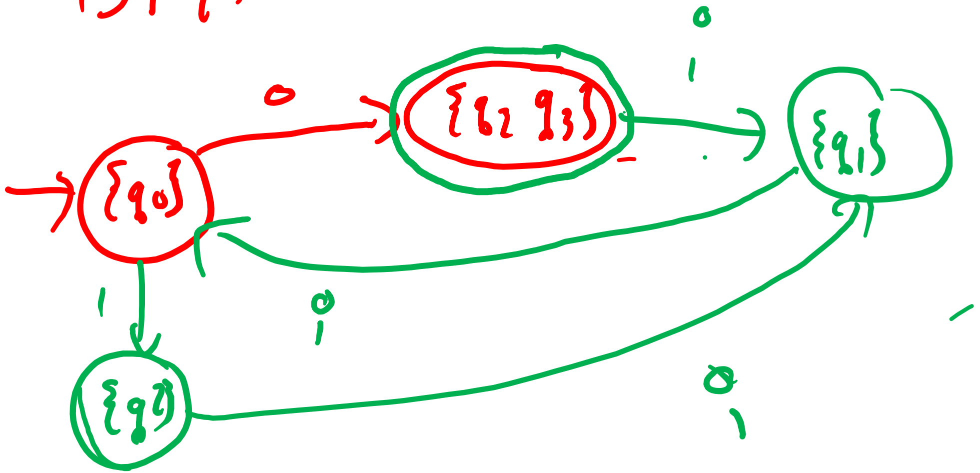
Types?

Subset construction examples

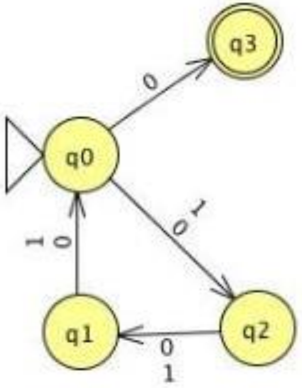


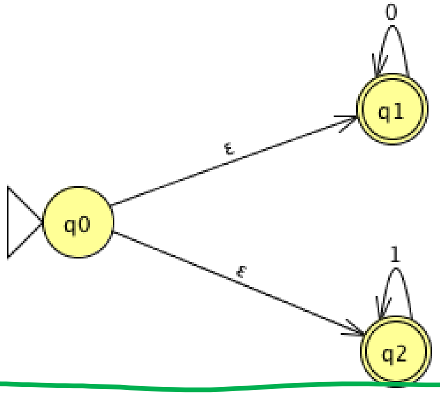
$2^4 = 16$

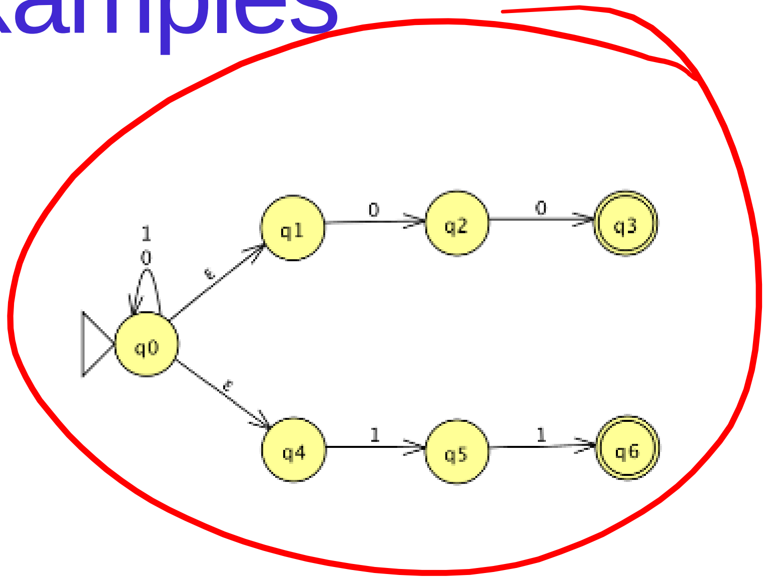
NFA to DFA

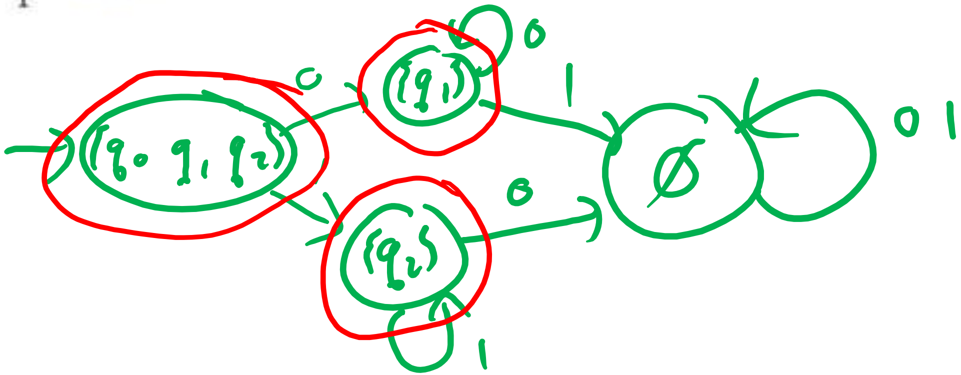


Subset construction examples









Exercise.

DFA equiv NFA

Theorem: For each language L,

L is recognizable by some DFA

iff

L is recognizable by some NFA

DFA equiv NFA equiv RegExp

Theorem: For each language L,

L is recognizable by some DFA

iff

L is recognizable by some NFA

iff

L is describable by some regular expression

For next time

- Work on Individual HW2 **due Tuesday**

Pre class-reading for Wednesday: Example 1.56 on page 58