

CSE 105

THEORY OF COMPUTATION

"Winter" 2018

<http://cseweb.ucsd.edu/classes/wi18/cse105-ab/>

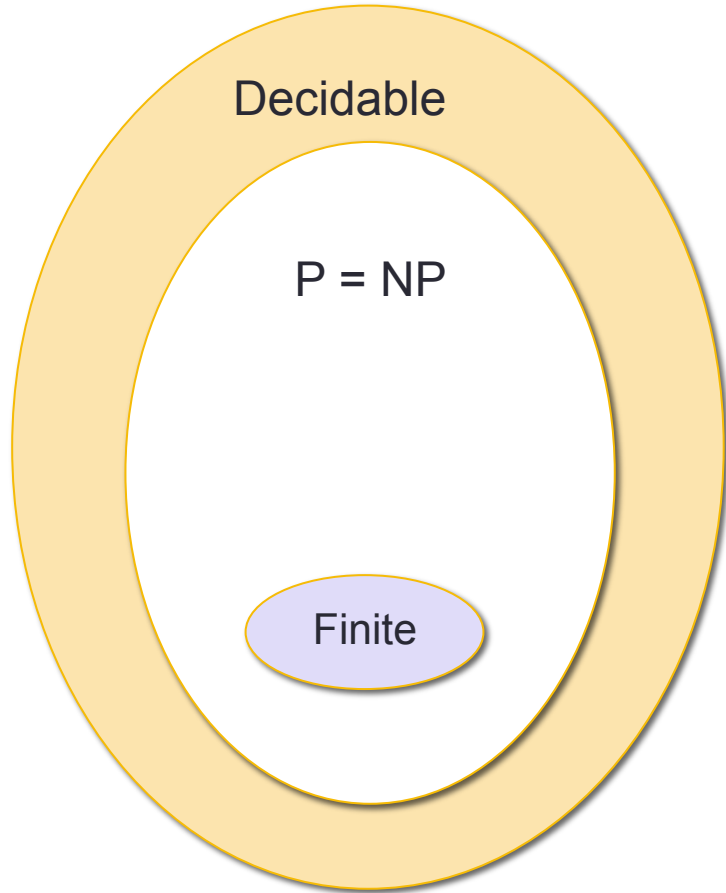
Today's learning goals

Sipser Ch 7

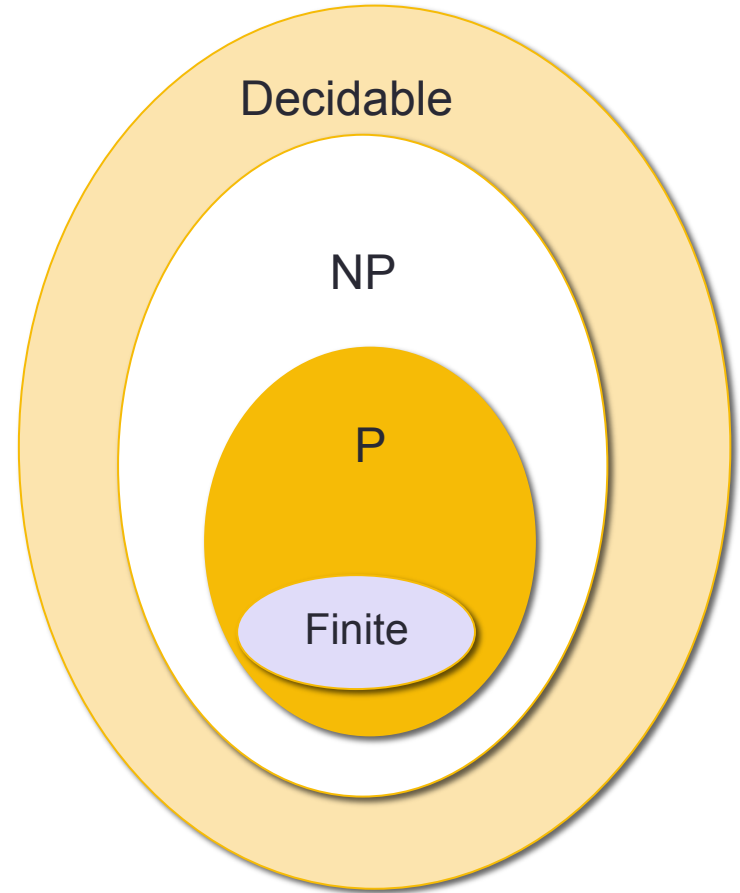
- Define NP-completeness
- Give examples of NP-complete problems
- Use polynomial-time reduction to prove NP-completeness

- Section 7.4, 7.5: NP-completeness

Start review!



or



P vs. NP

Problems in P	Problems in NP
(Membership in any) regular language	Any problem in P
(Membership in any) CFL	HAMPATH
PATH	CLIQUE
A_{DFA}	VERTEX-COVER
E_{DFA}	TSP
EQ_{DFA}	SAT
Addition, multiplication of integers	...

How to answer $P = NP$?

Are there **hardest** NP problems?

Reductions to the rescue

Sipser p. 299-305

1970s Stephen Cook and Leonid Levin **independently and in parallel** lay foundations of **NP-completeness**

Definition A language B is **NP-complete** if (1) it is in NP and (2) every A in NP is polynomial-time reducible to it.

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Sipser p. 299-305

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Consequence If an NP-complete problem has a polynomial time solution then **all** NP problems are polynomial time solvable.

Reductions to

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Definition A language B
(2) every A in NP is polyr

What would prove that $P = NP$?

- A. Showing that a problem solvable by brute-force methods has a nondeterministic solution.
- B. Showing that there are two distinct NP-complete problems.
- C. Finding a polynomial time solution for an NP-complete problem.
- D. Proving that an NP-complete problem is not solvable in polynomial time.
- E. I don't know

Consequence If an NP-complete problem has a polynomial time solution then **all** NP problems are polynomial time solvable

3-SAT

Rosen p. 299

Cook-Levin Theorem: 3-SAT is NP-complete.

$$(x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge (x \vee y \vee z)$$



Are other problems NP complete?

To prove that X is NP-complete

From scratch: Prove it is NP, and that all NP problems are polynomial-time reducible to it.

Using reduction: Show that a (known-to-be) NP complete problem reduces to it.

3SAT polynomial-time reduces to CLIQUE

Sipser p. 302

Given: Boolean formula in CNF with exactly 3 literals/clause

- AND of ORs

- args in OR clauses: var or negated var

Desired Answer: Yes if satisfiable; No if unsatisfiable

Instead transform formula to graph so that *graph has clique iff original formula is satisfiable*

3SAT polynomial-time reduces to CLIQUE

Transform 3-CNF formula with k clauses to graph G

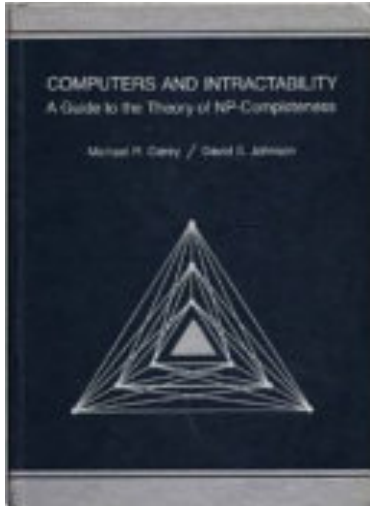
- vertices are the literals in each clause
- edges between all vertices except
 - two literals in the same clause
 - literals that are negations of one another

Claim: formula is satisfiable iff G has k -clique

3-SAT to Clique example

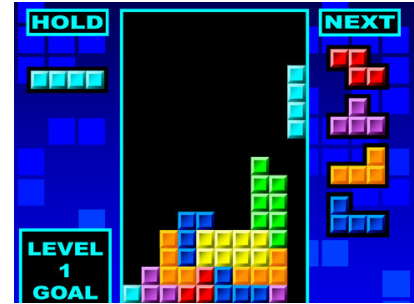
$$(x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge (x \vee y \vee z)$$

Are other problems NP-complete?



MY HOBBY:
EMBEDDING NP-COMPLETE PROBLEMS IN RESTAURANT ORDERS

CHOTCHKIES RESTAURANT	
APPETIZERS	
MIXED FRUIT	2.15
FRENCH FRIES	2.75
SIDE SALAD	3.35
HOT WINGS	3.55
MOZZARELLA STICKS	4.20
SAMPLER PLATE	5.80
SANDWICHES	
BARBECUE	6.55



Next time

Review for final exam

Please fill out CAPE, TA evaluations.

Seat charts for final exam on Piazza.

Lecture B: PCYNH109