CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/

Today's learning goals

Sipser Ch 4.2

- Trace high-level descriptions of algorithms for computational problems.
- Use counting arguments to prove the existence of unrecognizable (undecidable) languages.
- Use diagonalization in a proof of undecidability.

Reminder - Exam 2 next class (Friday)

* Review session tonight (w/ podcast)
* Exam 2 practice Q solutions to be posted tmrw
* Seat assignments on Piazza

Computational problems

A computational problem is **decidable** iff the language encoding the problem instances is decidable.

In Siper 4.1: The computational problems below

A_{DFA}, A_{NFA}, A_{REX}, A_{CFG}
E_{DFA}, E_{NFA}, E_{REX}, E_{CFG}
EQ_{DFA}, EQ_{NFA}, EQ_{REX}, EQ_{CFG}
are all decidable

Undecidable?

High level TM
- L(M)
- decider

- There are many ways to prove that a problem ***is*** decidable.

use prev algorithms

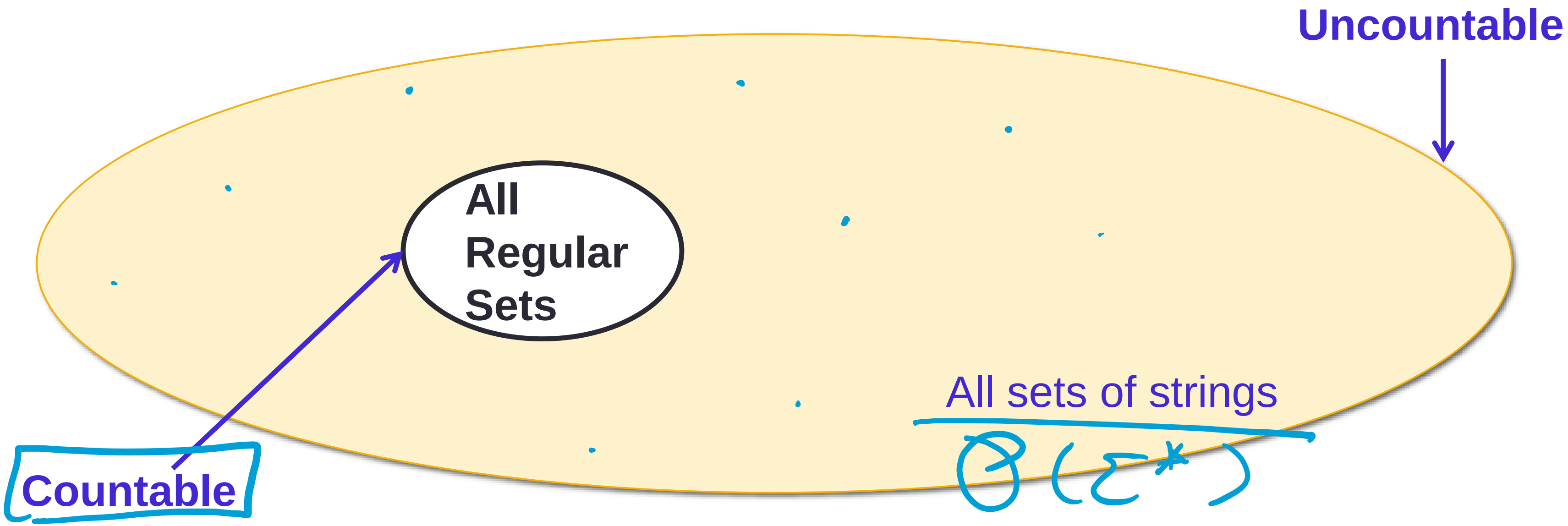
- How do we find (and prove) that a problem ***is not*** decidable?

No alg can decide problem

Counting arguments

Before we proved the Pumping Lemma …

We proved there was a set that was not regular because

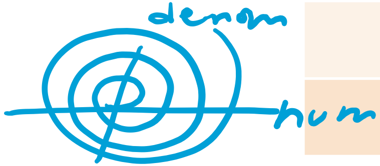


Reminder: countable/ uncountable

Sets A and B have the **same size** $|A| = |B|$ means there is a function between them that is both one-to-one and onto.

1 2 3 4 5 . . . .

0 1 -1 2 -3



Countable (finite or same size as N)	Uncountable
N, Z, Q	R
Σ^*	{ infinite sequences over Σ}
The set of all TMs	$P(\Sigma^*)$

e.g. $\Sigma = \{0, 1\}$

ε, 0, 1, 00, 01, 10, 11, 000, 001, . . —

Sipser p. 202-204

Reminder: countable/ uncountable

Sets A and B have the **same size** $|A| = |B|$ means there is a function between them that is both one-to-one and onto.

What type of elements are in the set $P(\Sigma^*)$?

~~A.~~ Strings Σ^*

~~B.~~ Regular expressions

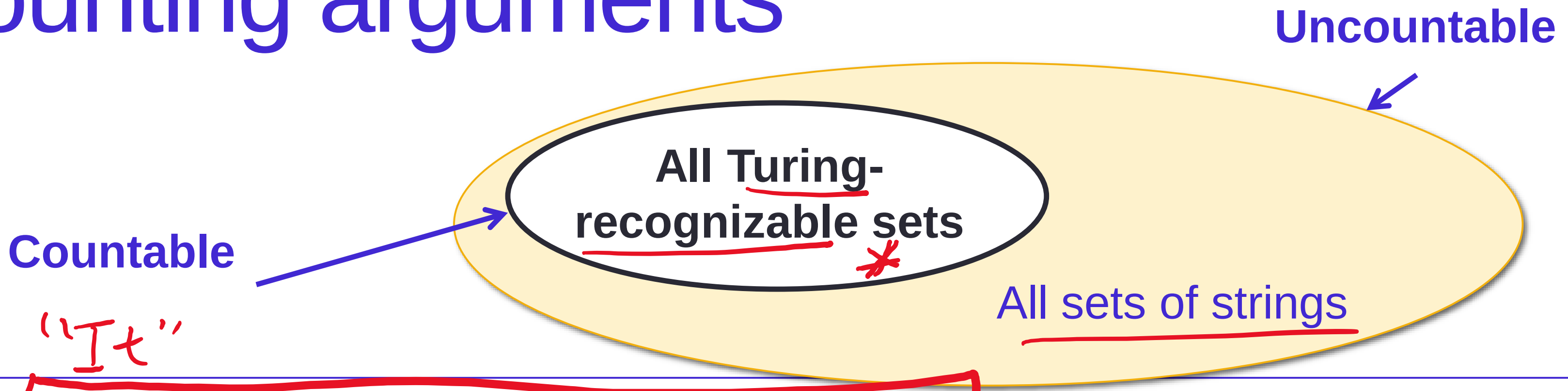
C. Languages Sets of strings

~~D.~~ Sets of regular expressions

E. I don't know

Sipser p. 202-204

Counting arguments



"It"

Why is the set of Turing-recognizable languages **countable**?

A. It's equal to the set of all TMs, which we showed is countable.

B. It's a subset of the set of all TMs, which we showed is countable.

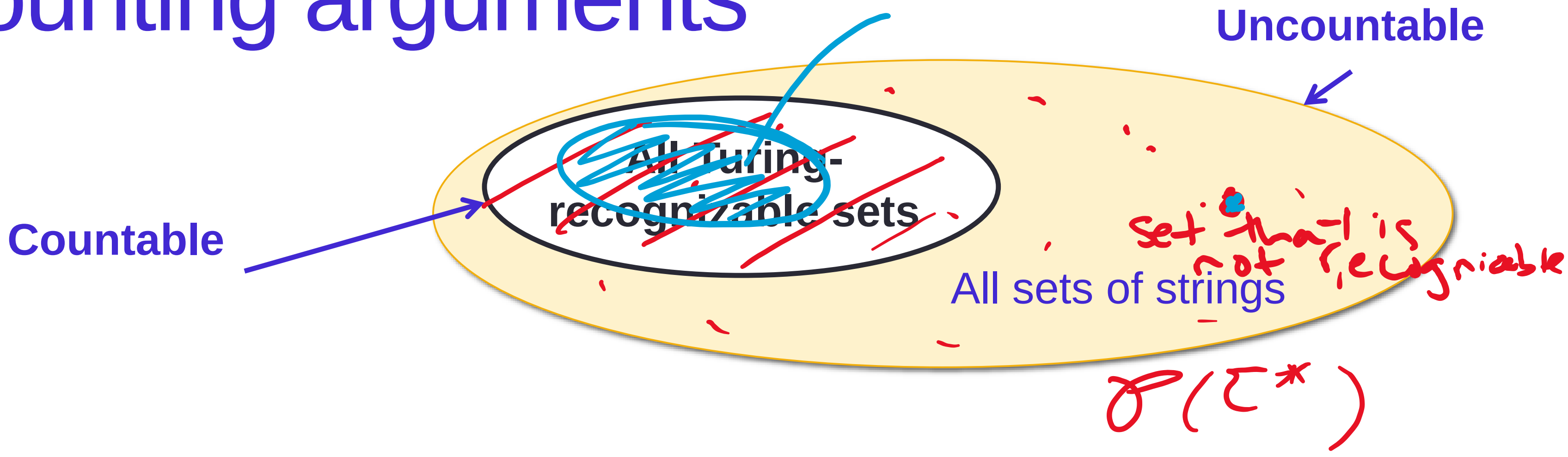
C. Each Turing-recognizable language is associated with a TM, so there can be no more Turing-recognizable languages than TMs.

D. More than one of the above.

E. I don't know.

Counting arguments

Turing-dec



Is the set of Turing-decidable sets countable?

$$\{\text{Turing-dec}\} \subseteq \{\text{Turing-rec}\}$$

Satisfied?

- Maybe not …
- What's a specific example of a language that is not Turing-recognizable? or not Turing-decidable?

Cantor's diagonalization

- *Idea: consider a set that, were it to be Turing-decidable, would have to "talk" about itself, and contradict itself!*

there is a TM that decides this set

A_{TM}

Recall A_{DFA} = {<B,w> | B is a DFA and w is in L(B) }

A_{TM} = {<M,w> | M is a TM and w is in L(M) }

What is A_{TM} ?

A. A Turing machine whose input is codes of TMs and strings.

B. A set of pairs of TMs and strings.

C. A set of strings that encode TMs and strings.

D. Not well defined.

E. I don't know.

A_{TM}

$L(N) = A_{TM}$

$A_{TM} = \{\langle M,w \rangle \mid M \text{ is a TM and } w \text{ is in } L(M)\}$

Define the TM N = "On input $\langle M,w \rangle$:

string codes description M and w

1. Simulate M on w.
2. If M accepts, accept. If M rejects, reject."

Which of the following statements is true?

~~A. N decides A_{TM}~~

B. N recognizes A_{TM}

~~C. N always halts~~

D. More than one of the above.

E. I don't know

M may not be decider so step 1 of N may loop b/c M may loop on w

A_{TM}

A_{DFA} = {<B,w> | B is a DFA and w is in L(B) }

Decider for this set simulates arbitrary DFA

A_{TM} = {<M,w> | M is a TM and w is in L(M) }

Decider for this set simulates arbitrary TMs.

What happens when it simulates itself?

WTS uh oh

Diagonalization proof: A_{TM} not decidable Sipser p. 207

Assume, towards a contradiction, that it is.

always halts

not a decider

Call M_{ATM} the decider for A_{TM}:

$M_{ATM} \neq N$

For every TM M and every string w,

- Computation of M_{ATM} on <M,w> halts and accepts if w is in L(M).
- Computation of M_{ATM} on <M,w> halts and rejects if w is not in L(M).

Goal: find contradiction

Diagonalization proof: A_{TM} not decidable *Sipser 4.11*

Assume, towards a contradiction, that M_{ATM} decides A_{TM}

A TM

Define the TM D = "On input <M>:

1. Run M_{ATM} on <M, <M>>. using assumption
2. If M_{ATM} accepts, reject; if M_{ATM} rejects, accept."

D on <D> ?

Diagonalization proof: A_{TM} not decidable *Sipser 4.11*

Assume, towards a contradiction, that M_{ATM} decides A_{TM}

Define the TM D = "On input <M>:

1. Run M_{ATM} on <M, <M>>.
2. If M_{ATM} accepts, reject; if M_{ATM} rejects, accept."

Which of the following computations halt?

A. Computation of D on <X> where X is TM which accepts string w if first character is 0 and loops otherwise.

B. Computation of D on <Y> where Y is TM with L(Y) =Σ^*

C. Computation of D on <D>

D. All of the above.

Diagonalization proof: A_{TM} not decidable *Sipser 4.11*

Assume, towards a contradiction, that M_{ATM} decides A_{TM}

Define the TM D = "On input <M>:

1. Run $\boxed{M_{ATM}}$ on <M, <M>>. * M_{ATM} is a decider so halts
2. If M_{ATM} accepts, reject; if M_{ATM} rejects, accept." * 1 step

Consider **running D on input <D>**. Because D is a decider:

- **either computation halts and accepts …**
- **or computation halts and rejects …**

Case ① D accepts $\langle D \rangle$

Running D: in step 1 M_{ATM} gets input $\langle D, \langle D \rangle \rangle$ and accepts because $\langle D \rangle \in L(D)$ so in step 2 D rejects $\langle D \rangle$ $\rightarrow\leftarrow$

Case ② D rejects $\langle D \rangle$

Running D: in step 1, M_{ATM} gets input $\langle D, \langle D \rangle \rangle$ and rejects because $\langle D \rangle \notin L(D)$ so in step 2 D accepts $\langle D \rangle$ $\rightarrow\leftarrow$

Diagonalization proof: A_{TM} not decidable *Sipser 4.11*

Assume, t

Def

1. R
2. If

Diagonalization???

Self-reference

"Is <D> an element of L(D)?"

Consider **running** **on input <D>**. Because D is a decider:

- **either comp ation halts and accepts …**
- **or computation halts and rejects …**

A_{TM}

- Recognizable
- Not decidable

Fact (from discussion section): A language is decidable iff it and its complement are both recgonizable.

Corollary: The complement of A_{TM} is **unrecognizable**.

Do we have to diagonalize?

- Next time (after exam): comparing difficulty of problems.

Next time: Exam 2

Bring ID, note card

Check seat maps before coming to class