CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/

Today's learning goals

Sipser Ch 4.1

- Explain what it means for a problem to be decidable.
- Justify the use of encoding.
- Give examples of decidable problems.

Computational problems

A computational problem is **decidable** iff the language encoding the problem instances is decidable.

We won't specify the encoding.

To prove decidable, define TM M = "On input <…>,

1.

2. … "

Show (1) L(M) = … and (2) M is a decider.

Computational problems over Σ

A_{DFA} "Is a given string accepted by a given DFA?"

{ <B,w> | B is a DFA, w in Σ*, and w is in L(B) }

E_{DFA} "Is the language of a DFA empty?"

{ <A> | A is a DFA over Σ, L(A) is empty }

EQ_{DFA} "Are the languages of two given DFAs equal?"

{ <A, B> | A and B are DFA over Σ, L(A) = L(B) }

From last week

A_{DFA}

M_1 = "On input <B,w>, where B is a DFA and w is a string:

1. Simulate B on input w (by keeping track of states in B, transition function of B, etc.)
2. If the simulations ends in an accept state of B, *accept*. If it ends in a non-accept state of B, *reject*. "

E_{DFA}

M_2 = "On input <A>, where A is a DFA:

1. Mark the start state of A.
2. Repeat until no new states get marked:
   i. Loop over states of A and mark any unmarked state that has an **incoming** edge from a marked state.
3. If no final state of A is marked, *accept*; otherwise, *reject*."

Non-emptiness?

E'_{DFA} "Is the language of a DFA non-empty?"

Is this problem decidable?

A. Yes, using M_3 in the handout.
B. Yes, using M_4 in the handout.
C. Yes, both M_3 and M_4 work.
D. Yes, but not using the machines in the handout.
E. No.

M_3 and M_4 both recognize E'_{DFA}

M_3 is a decider.
M_4 is not a decider.

Proving decidability

Claim: EQ_{DFA} is decidable

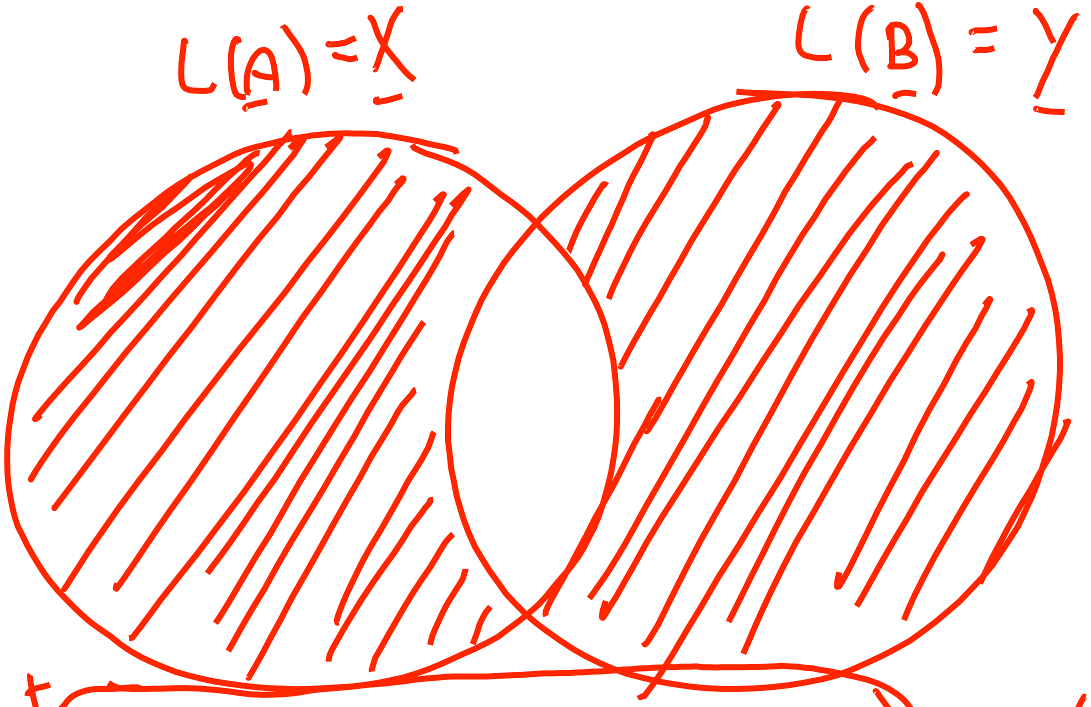
Proof: WTS that { <A, B> | A, B are DFA over Σ, L(A) = L(B) } is decidable. Idea: give high-level description

Step 1: construction

Will we be able to simulate A and B?

What does set equality mean?

Can we use our previous work?



Let C be a DFA that recognizes

$$(X \cap \bar{Y}) \cup (\bar{X} \cap Y) = \emptyset \iff X = Y$$

Proving decidability

Claim: EQ_{DFA} is decidable

Proof: WTS that { <A, B> | A, B are DFA over Σ, L(A) = L(B) } is decidable. Idea: give high-level description

Step 1: construction

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Proving decidability

Claim: EQ_{DFA} is decidable

Proof: WTS that { <A, B> | A, B are DFA over Σ, L(A) = L(B) } is decidable. Idea: give high-level description

Step 1: construction

$$X = Y \text{ iff } \left((X \cap Y^c) \cup (Y \cap X^c) \right) = \emptyset$$

Very high-level:

Build new DFA recognizing symmetric difference of A, B. Check if this set is empty.

Proving decidability

Claim: EQ_{DFA} is decidable

Proof: WTS that { <A, B> | A, B are DFA over Σ, L(A) = L(B) } is decidable. Idea: give high-level description

Step 1: construction

Define TM M_5 by: M_5 = "On input <A,B> where A,B DFAs:

1. Construct a new DFA, D, from A,B using algorithms for complementing, taking unions of regular languages such that L(D) = symmetric difference of A and B.
2. Run machine M_2 on <D>.
3. If it accepts, accept; if it rejects, reject."

Proving decidability

Step 1: construction

Define TM M_5 by: M_5 = "On input <A,B> where A,B DFAs

o.w. reject if not of the right type.

1. Construct a new DFA, D, from A,B using algorithms for complementing, taking unions of regular languages such that L(D) = symmetric difference of A and B.
2. Run machine M_2 on <D>.
3. If it accepts, accept; if it rejects, reject."

Step 2: correctness proof

WTS (1) $L(M_5) = EQ_{DFA}$ and (2) M_5 is a decider.

Proof: ① ⓐ $L(M_5) \subseteq EQ_{DFA}$. Let $w \in L(M_5)$

then $w = \langle A, B \rangle$ such that A and B are DFAs, and $L(D)$ is empty where D recognizes

$$\left(L(A) \cap \overline{L(B)}\right) \cup \left(\overline{L(A)} \cap L(B)\right).$$

Symmetric difference of $L(A)$, $L(B)$ is empty

$\Leftrightarrow L(A) = L(B)$

Exercise: Prove this.

$\Rightarrow \langle A, B \rangle \in EQ_{DFA}$

ⓑ $EQ_{DFA} \subseteq L(M_5)$ exercise.

② M_5 is a decider

Computational problems

Which of of the following computational problems are **decidable**?

A. A_{NFA}
B. E_{NFA}
C. EQ_{NFA}
D. All of the above
E. None of the above

Computational problems

Compare:

$E_{DFA} = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = \phi\}$

$E_{REX} = \{\langle R\rangle \mid R \text{ is a regular expression } L(R) = \phi\}$

A. $A_{REX} = A_{NFA} = A_{DFA}$, $E_{REX} = E_{NFA} = E_{DFA}$, $EQ_{REX} = EQ_{NFA} = EQ_{DFA}$

B. They're all decidable, some are equal and some not.

C. They're of different types so all are different.

D. None of the above

Techniques

Sipser 4.1

- **Subroutines**: can use decision procedures of decidable problems as subroutines in other algorithms
  - A_{DFA}
  - E_{DFA}
  - EQ_{DFA}
- **Constructions**: can use algorithms for constructions as subroutines in other algorithms
  - Converting DFA to DFA recognizing complement (or Kleene star).
  - Converting two DFA/NFA to one recognizing union (or intersection, concatenation).
  - Converting NFA to equivalent DFA.
  - Converting regular expression to equivalent NFA.
  - Converting DFA to equivalent regular expression.

Next time

- Are all computational problems decidable?

For Wednesday, pre-class reading: Section 4.3, page 207-209.

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