CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/

Today's learning goals

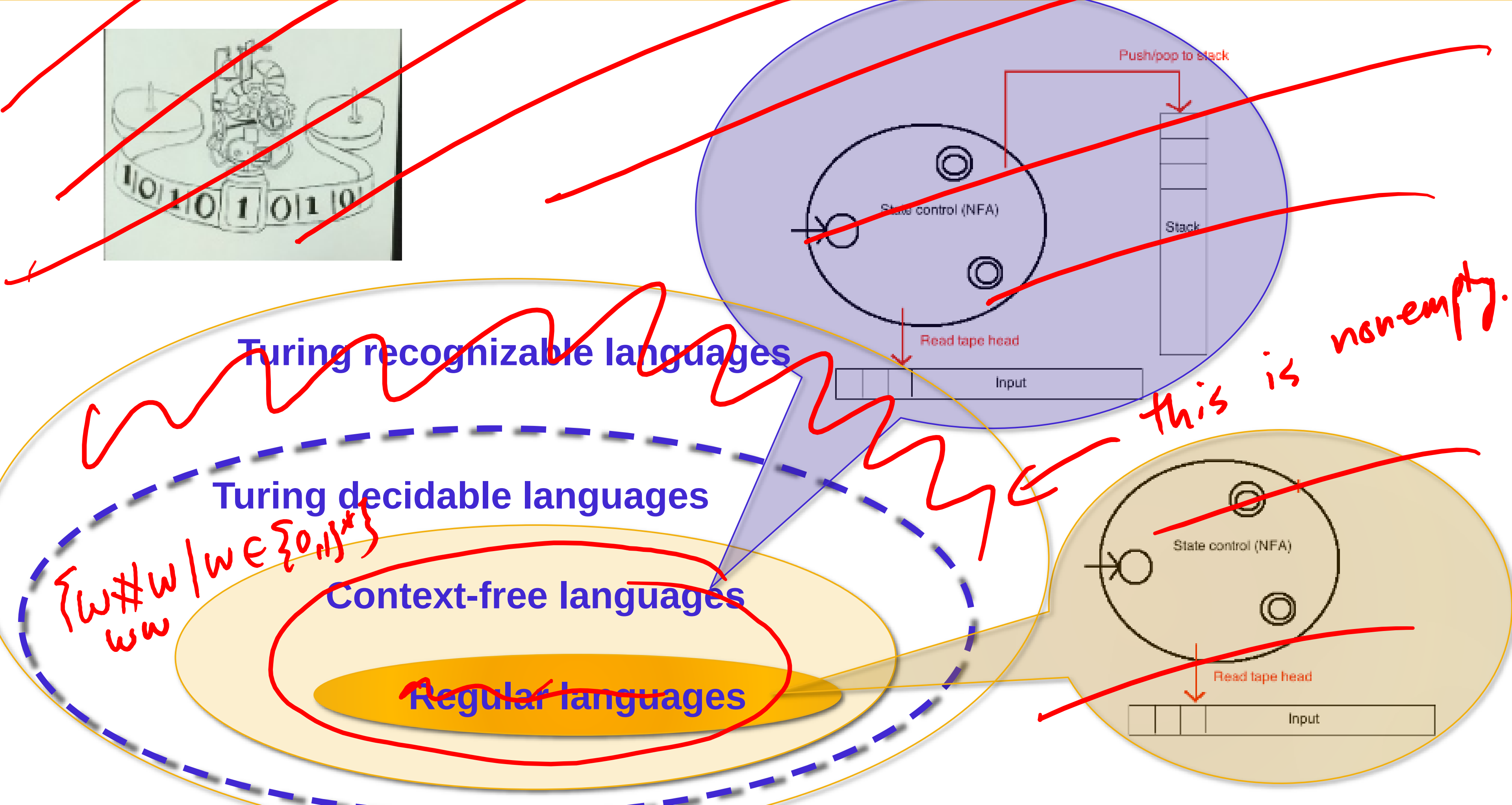
Sipser Section 3.2

- Describe several variants of Turing machines and informally explain why they are equally expressive.
- State and use the Church-Turing thesis.

Describing TMs

Sipser p. 184-185

- **Formal definition**: set of states, input alphabet, tape alphabet, transition function, state state, accept state, reject state.
- **Implementation-level definition**: English prose to describe Turing machine head movements relative to contents of tape.
- **High-level desciption**: Description of algorithm, without implementation details of machine. As part of this description, can "call" and run another TM as a subroutine.



1 0 1 0 1 0 1 0
Push/pop to stack
State control (NFA)
Stack
Read tape head
Input
Turing recognizable languages
Turing decidable languages
Context-free languages
Regular languages
{w#w | w ∈ {0,1}*}
ww
this is nonempty.
State control (NFA)
Read tape head
Input

High-level description = Algorithm

- Wikipedia "self-contained step-by-step set of operations to be performed"
- CSE 20 textbook "An algorithm is a finite sequence of precise instructions for performing a computation or for solving a problem."

Church-Turing thesis →

Each algorithm can be implemented by some Turing machine.

Variants of TMs

- Scratch work, copy input, …
- Parallel computation
- Printing vs. accepting
- More flexible transition function
  - Can "stay put" ✗
  - Can "get stuck"
  - Can "goto" cell on tape ✗
  - *lots of examples in exercises* to Cha

Multiple tapes

Nondeterminism

Enumerators

All these models are equally expressive!

Also: wildly different models

- **λ-calculus, Post canonical systems, URMs, etc.**

"Equally expressive"

Model 1 is **equally expressive** as Model 2 iff

- every language recognized by some machine in Model 1 is recognizable by some machine in Model 2**, and**
- every language recognized by some machine in Model 2 is recognizable by some machine in Model 1.

Which of the following statements is true?

$0^n 1^n$

A. NFAs and PDAs are equally expressive because they may both be nondeterministic.

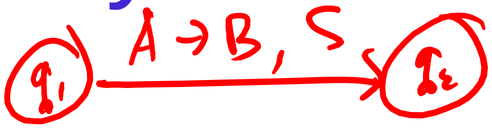
$w\#w$

B. PDAs and Turing machines are equally expressive because they can both write (to a stack or the tape).

C. NFAs and DFAs are equally expressive because they can be translated to one another.

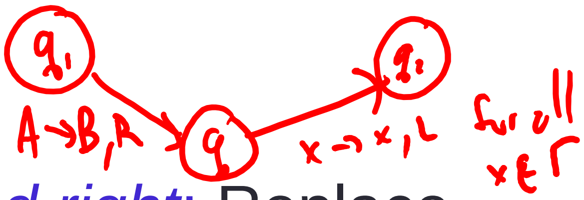
D. None of the above.

Turing machines that can stay



- Transition function

$$Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R,S\}$$



Sketch of proof of equivalence:

To allow for stay put instead of only left and right: Replace each stay put transition with two transitions: one that moves to the right and the second back to the left.

Sipser: 176

Multitape TMs

Sipser p. 176

- As part of construction of machine, declare some finite number of tapes that are available. k
- Input given on tape 1, rest of the tapes start blank.
- Each tape has its own read/write head.
- Transition function

$$Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L,R,S\}^k$$

Sketch of proof of equivalence:

To simulate multiple tapes with one tape: Use delimiter to keep tape contents separate, use special symbol to indicate location of each read/write head.

$(\sqcup,\sqcup,\sqcup,\sqcup)$

Nondeterministic TMs

Sipser p. 178

At any point in the computation, machine may proceed according to several possibilities.

Transition function

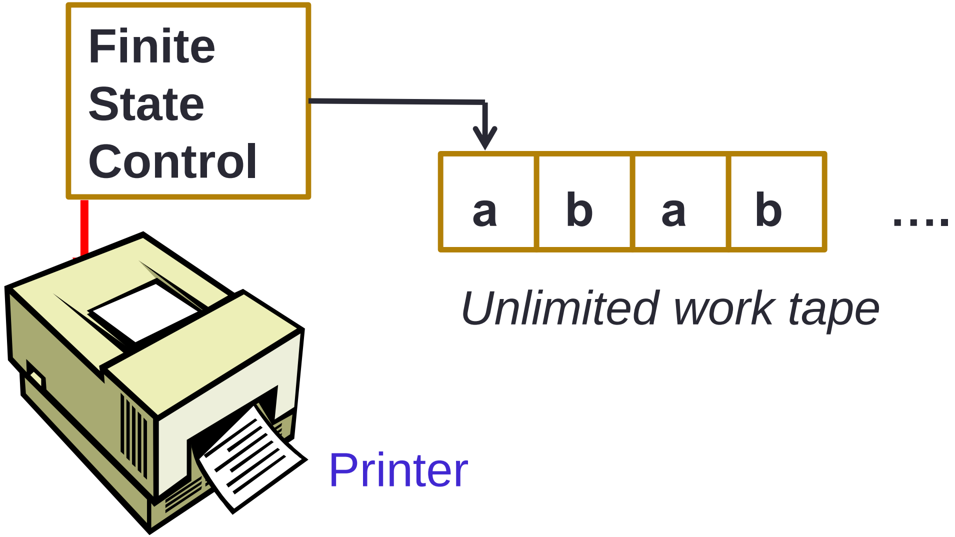
$$Q \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{L,R\})$$

Sketch of proof of equivalence:

To simulate nondeterministic machine: Use 3 tapes to do breadth-first search of computation tree: "read-only" input tape, simulation tape, tape tracking nondeterministic braching.

Very different model: Enumerators *Sipser p. 180*

Produce language as output rather than recognize input



Computation proceeds according to transition function.

At any point, machine may "send" a string to printer.

L(E) = { w | E eventually, in finite time, prints w}

Enumerators

Which of the following is a high level description for an enumerator that enumerates the set {0}?

A. "On input w, if w = 0 accept, otherwise reject."
B. "Ignore input. If w = 0 accept, otherwise reject."
C. "On input w, reject."
D. "Ignore input. Print the string 0."
E. None of the above.

Recognition and enumeration *Sipser Theorem 3.21*

Theorem: A language L is Turing-recognizable iff some enumerator enumerates L.

Proof:

1. Assume L is Turing-recognizable. WTS some enumerator enumerates it.
2. Assume L is enumerated by some enumerator. WTS L is Turing-recognizable.

Recognition and enumeration *Sipser Theorem 3.21*

2. Assume the enumerator E enumerates L. WTS L is Turing-recognizable.

We'll use E in a subroutine for high-level description of Turing machine M that will recognize L.

Define M as follows: M = "On input w,

1. Run E. For each string x printed by E
   - If x = w, accept. Otherwise, continue."

Correctness?

Recognition and enumeration *Sipser Theorem 3.21*

1. Assume L is Turing-recognizable. WTS some enumerator enumerates it.

Let M be a TM that recognizes L. We'll use M in a subroutine for high-level description of enumerator E.

Idea: check each string in turn to see if it is in L = L(M).

Standard string ordering: order strings first by length, then dictionary order. (p. 14)

Recognition and enumeration *Sipser Theorem 3.21*

1. Assume L is Turing-recognizable. WTS some enumerator enumerates it.

Let M be a TM that recognizes L. We'll use M in a subroutine for high-level description of enumerator E.

Let s_1, s_2, … be a list of all strings in Σ^* in standard string order

E = "Ignore any input. Repeat the following for i=1,2,3…

1. Run M for i steps on each input s_1, …, s_i
2. If any of the i computations of M accepts, print out the accepted string."

Correctness?

	Suppose M is TM that recognizes L	Suppose D is TM that decides L	Suppose E is enumerator that enumerates L
If string w is in L then …	accept		
If string w is not in L then …	reject or loop.		

For next time

GroupHW5 due Saturday, February 24

For Friday, pre-class reading: pp. 185 (middle)