CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/

Today's learning goals

Sipser Section 3.2

- Describe several variants of Turing machines and informally explain why they are equally expressive.
- State and use the Church-Turing thesis.

Describing TMs

Sipser p. 184-185

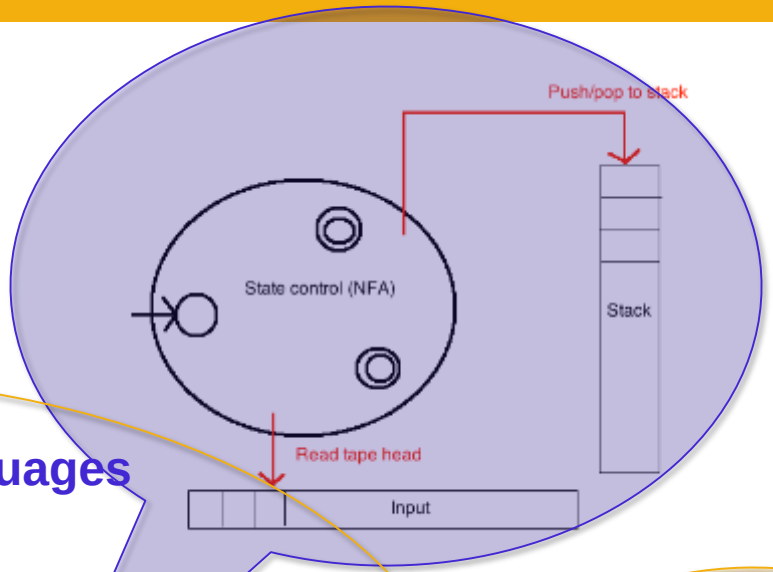
- **Formal definition**: set of states, input alphabet, tape alphabet, transition function, state state, accept state, reject state.
- **Implementation-level definition**: English prose to describe Turing machine head movements relative to contents of tape.
- **High-level desciption**: Description of algorithm, without implementation details of machine. As part of this description, can "call" and run another TM as a subroutine.

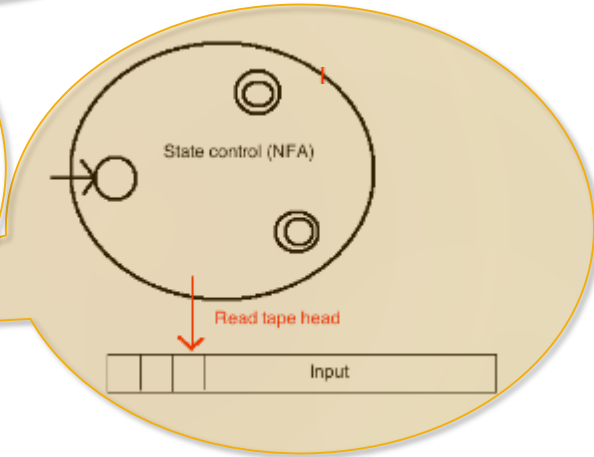
Turing recognizable languages

Turing decidable languages

Context-free languages

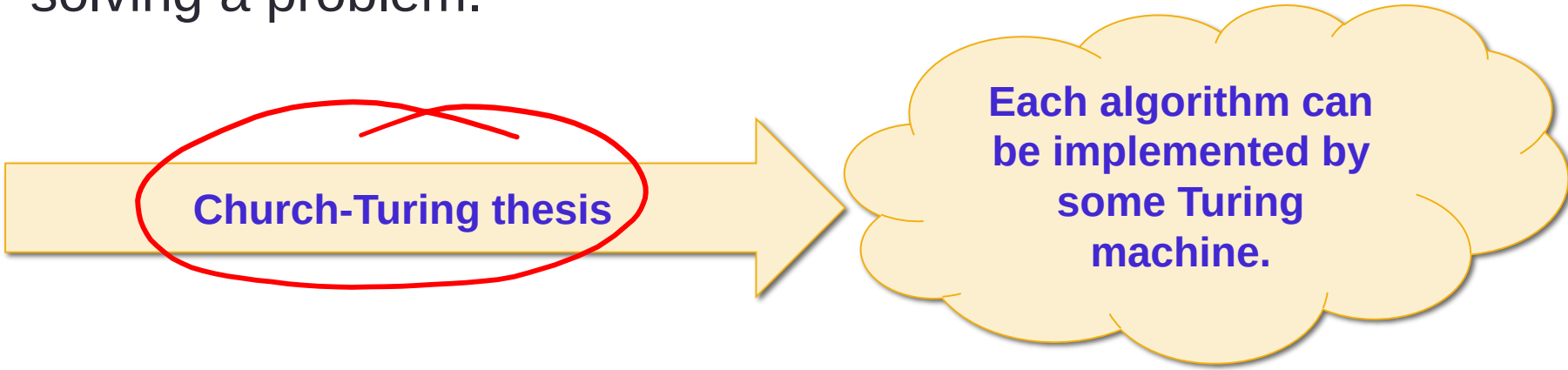
Regular languages





High-level description = Algorithm

- Wikipedia "self-contained step-by-step set of operations to be performed"
- CSE 20 textbook "An algorithm is a finite sequence of precise instructions for performing a computation or for solving a problem."



Variants of TMs

- Scratch work, copy input, …
- Parallel computation
- Printing vs. accepting
- More flexible transition function
  - Can "stay put"
  - Can "get stuck"
  - Can "goto" cell on tape
  - *lots of examples in exercises to Cha*

Multiple tapes

Nondeterminism

Enumerators

DFAs/NFAs equivalently expressible

DPDAs ≠ NPDAs

All these models are equally expressive!

Also: wildly different models

- **λ-calculus, Post canonical systems, URMs, etc.**

"Equally expressive"

Model 1 is **equally expressive** as Model 2 iff

- every language recognized by some machine in Model 1 is recognizable by some machine in Model 2**, and**
- every language recognized by some machine in Model 2 is recognizable by some machine in Model 1.

Which of the following statements is true?

A. NFAs and PDAs are equally expressive because they may both be nondeterministic.

B. PDAs and Turing machines are equally expressive because they can both write (to a stack or the tape).

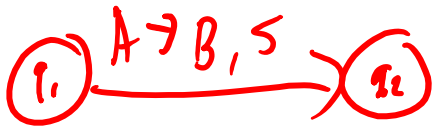
C. NFAs and DFAs are equally expressive because they can be translated to one another.

D. None of the above.

Turing machines that can stay

- Transition function

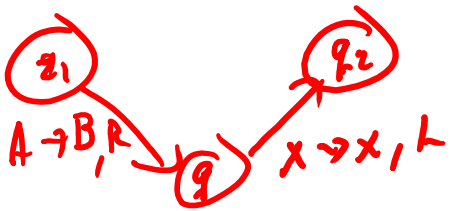
$$Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R,S\}$$



Sketch of proof of equivalence:

To allow for stay put instead of only left and right: Replace each stay put transition with two transitions: one that moves to the right and the second back to the left.

Sipser: 176



Multitape TMs

Sipser p. 176

- As part of construction of machine, declare some finite number of tapes that are available.
- Input given on tape 1, rest of the tapes start blank.
- Each tape has its own read/write head.
- Transition function

$$Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L,R,S\}^k$$

Sketch of proof of equivalence:

To simulate multiple tapes with one tape: Use delimiter to keep tape contents separate, use special symbol to indicate location of each read/write head.

Nondeterministic TMs

Sipser p. 178

At any point in the computation, machine may proceed according to several possibilities.

Transition function

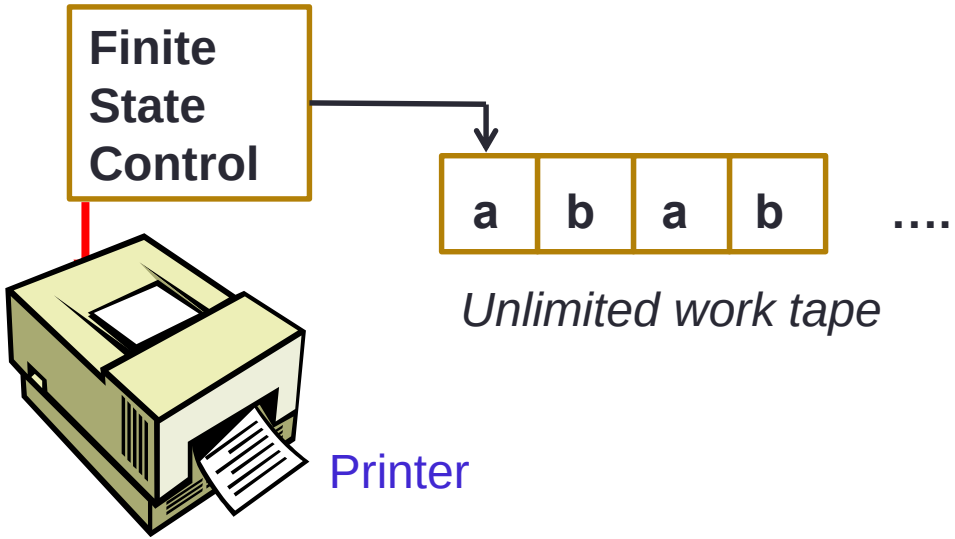
$$Q \times \Gamma \rightarrow P(Q \times \Gamma \times \{L,R\})$$

Sketch of proof of equivalence:

To simulate nondeterministic machine: Use 3 tapes to do breadth-first search of computation tree: "read-only" input tape, simulation tape, tape tracking nondeterministic braching.

Very different model: Enumerators *Sipser p. 180*

Produce language as output rather than recognize input



Computation proceeds according to transition function.

At any point, machine may "send" a string to printer.

L(E) = { w | E eventually, in finite time, prints w}

Enumerators

Which of the following is a high level description for an enumerator that enumerates the set {0}?

A. "On input w, if w = 0 accept, otherwise reject."
B. "Ignore input. If w = 0 accept, otherwise reject."
C. "On input w, reject."
D. "Ignore input. Print the string 0."
E. None of the above.

Recognition and enumeration *Sipser Theorem 3.21*

Theorem: A language L is Turing-recognizable iff some enumerator enumerates L.

Proof:

1. Assume L is Turing-recognizable. WTS some enumerator enumerates it.
2. Assume L is enumerated by some enumerator. WTS L is Turing-recognizable.

Recognition and enumeration *Sipser Theorem 3.21*

2. Assume the enumerator E enumerates L. WTS L is Turing-recognizable.

We'll use E in a subroutine for high-level description of Turing machine M that will recognize L.

Define M as follows: M = "On input w,

1. Run E. For each string x printed by E
   - If x = w, accept. Otherwise, continue."

Correctness?

Recognition and enumeration *Sipser Theorem 3.21*

1. Assume L is Turing-recognizable. WTS some enumerator enumerates it.

Let M be a TM that recognizes L. We'll use M in a subroutine for high-level description of enumerator E.

Idea: check each string in turn to see if it is in L = L(M).

Standard string ordering: order strings first by length, then dictionary order. (p. 14)

Recognition and enumeration *Sipser Theorem 3.21*

1. Assume L is Turing-recognizable. WTS some enumerator enumerates it.

Let M be a TM that recognizes L. We'll use M in a subroutine for high-level description of enumerator E.

Let $s_1, s_2, \ldots$ be a list of all strings in Σ^* in standard string order

E = "Ignore any input. Repeat the following for i=1,2,3…

1. Run M for i steps on each input $s_1, \ldots, s_i$
2. If any of the i computations of M accepts, print out the accepted string."

Correctness?

	Suppose M is TM that recognizes L	Suppose D is TM that decides L	Suppose E is enumerator that enumerates L
If string w is in L then …			
If string w is not in L then …			

For next time

GroupHW5 due Saturday, February 24

For Friday, pre-class reading: pp. 185 (middle)