CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/

Today's learning goals

Sipser Section 3.1

- Trace the computation of a Turing machine on given input
- Describe the language recognized by a Turing machine
- Give an implementation-level description of a Turing machine
- Determine if a Turing machine is a decider

(Group HW4 : Ch2 - due Sat)

Turing machine computation

- Read/write head starts at leftmost position on tape
- Input string written on leftmost squares of tape, rest is blank
- Computation proceeds according to transition function:
  - Given current state of machine, and current symbol being read
  - the machine
    - transitions to new state
    - writes a symbol to its current position (overwriting existing symbol)
    - moves the tape head L or R
- Computation ends **if and when** machine enters either the **accept** or the **reject** state.

Language of a Turing machine

L(M) = { w | computation of M on w halts after entering the **accept** state}

i.e. L(M) = { w | w is accepted by M}

Language of a Turing machine

Sipser p. 170

$$L(M) = \{ w \mid \underline{M \text{ accepts } w} \}$$

Which of the following is not always true?

A. If w is in L(M) then the computation of M on w halts and accepts.

B. If the computation of M on w halts and rejects, then w is not in L(M).

C. If w is not in L(M) then the computation of M on w halts and rejects.

Different from other models!

Computation of a Turing machine

To trace DFAs: enough to list states.
To trace NFAs: tree of possible current states (incl. spontaneous moves)
To trace PDAs: tree of possible computations incl. state + stack

- Current state ✓
- Current tape contents up to (finite) point after which all blank
- Current location of read/write head ✓

$u\ q\ v$

current state is q
current tape contents are uv (and then all blanks)
current head location is first symbol of v

Special configurations

For input string w

at leftmost cell

- Starting configuration q_0 w
- Accepting configuration u q_{acc} v
- Rejecting configuration u q_{rej} v

Halting configurations

current state is q
current tape contents are uv (and then all blanks)
current head location is first symbol of v

Language of a TM

Sipser p. 169

$L(M) = \{ w \mid M \text{ accepts } w\}$

$= \{ w \mid$ there is a sequence of configurations of M

where C_1 is start configuration of M on input w,

each C_i yields C_{i+1} and C_k is accepting configuration$\}$

computation of M on w

accepts

"The language of M"

"The language recognized by M"

Language of a TM

Sipser p. 170

- L is **recognized** by M if L = L(M)
- L is **decided** by M is L = L(M) and each computation of M halts, i.e. enters a halting configuration in finite time.

Challenge: *Convert the TM on the handout to one that recognizes the same language but does not decide it.*

{ww | w is in {0,1}* }

non context-free

An example

$$L = \{ w\#w \mid w \text{ is in } \{0,1\}^* \}$$

We already know that L is

- not regular
- not context-free

We will prove that L is

the language of some Turing machine

(and even is decided by Turing machine)

Implementation-level description

$$L = \{ w\#w \mid w \text{ is in } \{0,1\}^* \}$$

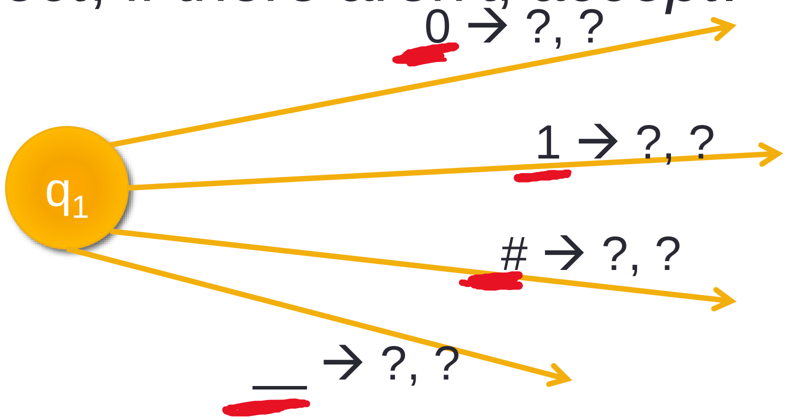
Idea for Turing machine

- Zig-zag across tape to corresponding positions on either side of '#' to check whether these positions agree. If they do not, or if there is no '#', *reject*. If they do, cross them off.
- Once all symbols to the left of the '#' are crossed off, check for any un-crossed-off symbols to the right of '#': if there are any, *reject*; if there aren't, *accept*.

Implementation-level description

Zig-zag across tape to corresponding positions on either side of '#' to check whether these positions agree. If they do not, or if there is no '#', *reject*. If they do, cross them off.

Once all symbols to the left of the '#' are crossed off, check for any un-crossed-off symbols to the right of '#': if there are any, *reject*; if there aren't, *accept*.

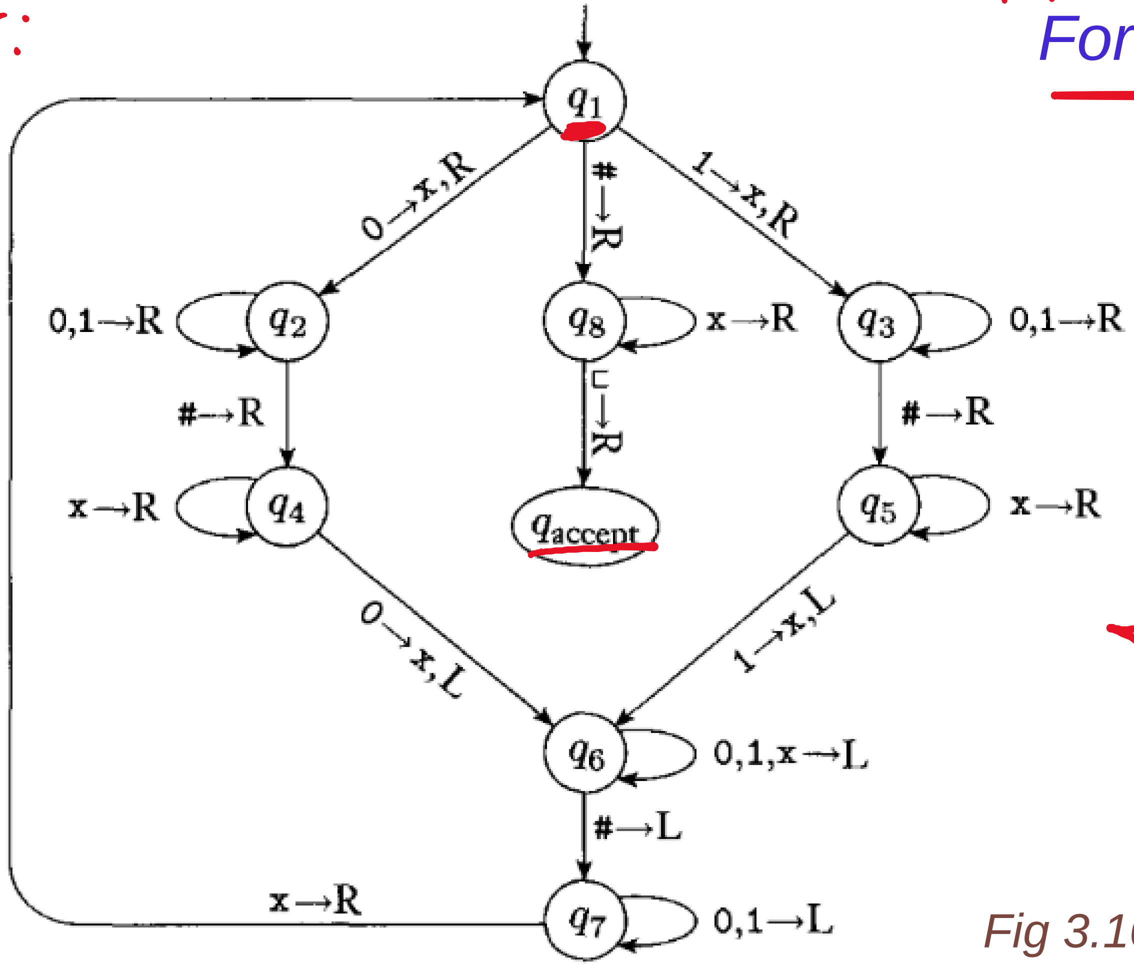


State diagram

TM (Q, Σ, Γ, δ, q_1, q_{accept}, q_{reject})

δ:

Formal definition



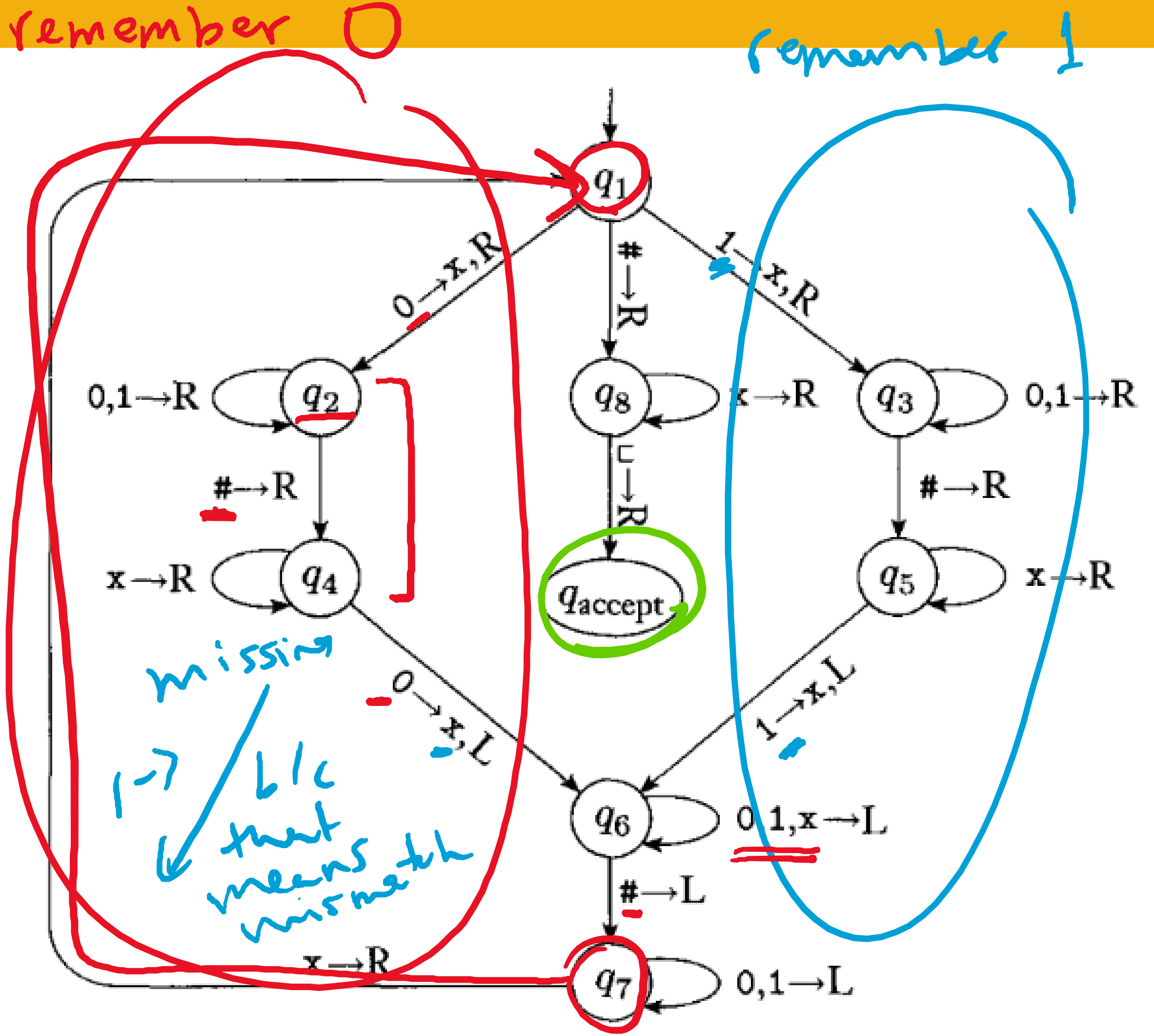
Q={q1,q2,q3,q4,q5,q6,q7,q8,q_{accept},q_{reject}}

Σ = {0,1,#}

Γ = {0,1,#,x,_ }

All missing transitions have output (q_{reject}, _, R)

Fig 3.10 in Sipser



Computation on input 0#0?

$q_1\ 0\#0$

$X\ q_2\ \#0$

$X\#\ q_4\ 0$

$X\ q_6\ \#X$

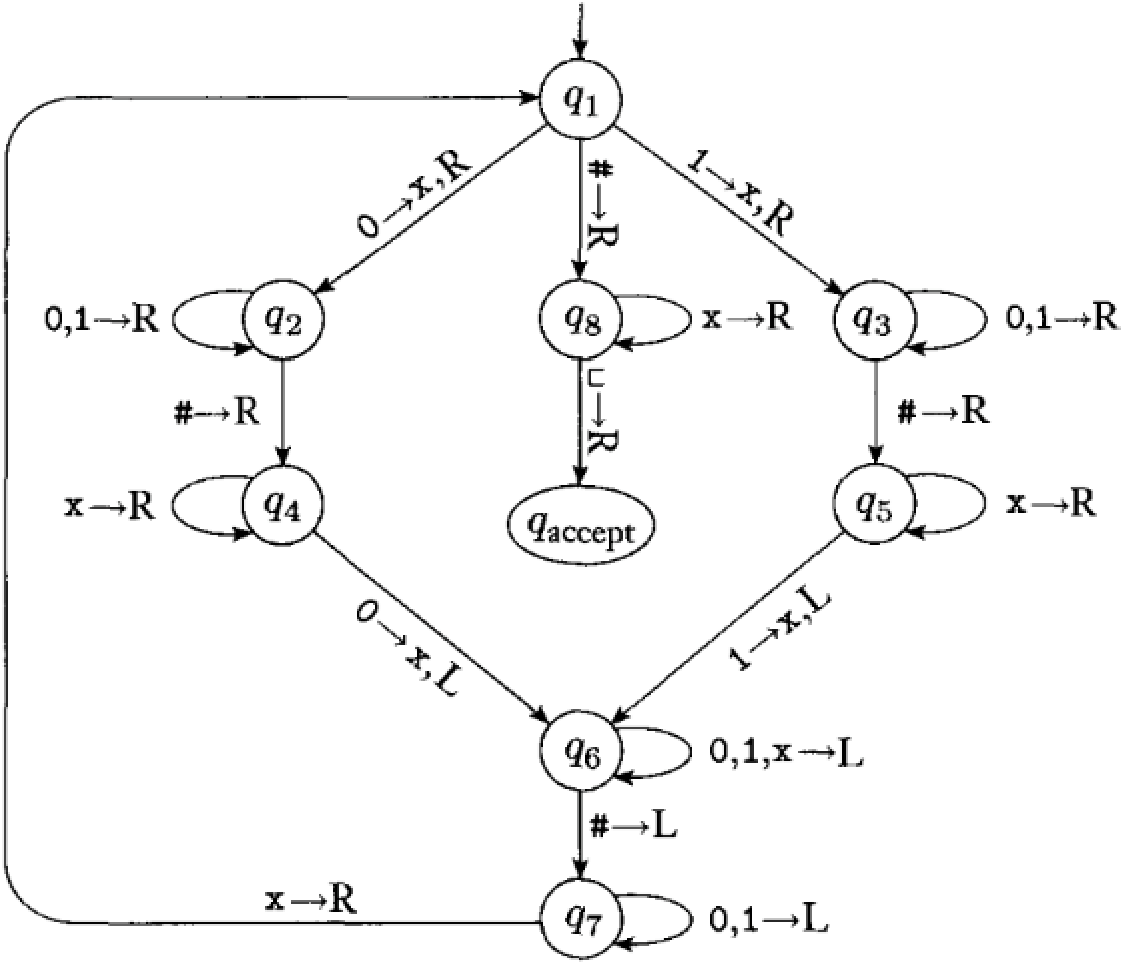
$q_7\ X\#X$

$X\ q_1\ \#X$

$X\ q_8\ X$

$XX\ q_{accept}$

Computation on input 0# ?



For next time

GroupHW4 due Saturday, February 17

For Friday, pre-class reading: pp. 184-185