CSE 105
THEORY OF COMPUTATION

"Winter" 2018

http://cseweb.ucsd.edu/classes/wi18/cse105-ab/

Today's learning goals

Sipser Section 3.1

- Trace the computation of a Turing machine on given input
- Describe the language recognized by a Turing machine
- Determine if a Turing machine is a decider
- Give an implementation-level description of a Turing machine

Turing machine computation

- Read/write head starts at leftmost position on tape
- Input string written on leftmost squares of tape, rest is blank
- Computation proceeds according to transition function:
  - Given current state of machine, and current symbol being read
  - the machine
    - transitions to new state
    - writes a symbol to its current position (overwriting existing symbol)
    - moves the tape head L or R
- Computation ends **if and when** machine enters either the **accept** or the **reject** state.

Language of a Turing machine

L(M) = { w | computation of M on w halts after entering the **accept** state}

i.e. L(M) = { w | w is accepted by M}

Language of a TM

Sipser p. 144

L(M) = { w | M accepts w}

Which of the following is not always true?

A. If w is in L(M) then the computation of M on w halts and accepts.
B. If the computation of M on w halts and rejects, then w is not in L(M).
C. If w is not in L(M) then the computation of M on w halts and rejects. or loops forever. . . . .

Computation

To trace DFAs: enough to list states.
To trace NFAs: tree of possible current states (incl. spontaneous moves)
To trace PDAs: tree of possible computations incl. state + stack

- Current state
- Current tape contents up to (finite) point after which all blank
- Current location of read/write head

u q v

current state is q
current tape contents are uv (and then all blanks)
current head location is first symbol of v

Special configurations

For input string w

- Starting configuration q_0 w
- Accepting configuration u q_{acc} v
- Rejecting configuration u q_{rej} v

} Halting configurations

current state is q
current tape contents are uv (and then all blanks)
current head location is first symbol of v

Language of a TM

Sipser p. 144

L(M) = { w | M accepts w}

= { w | there is a sequence of configurations $C_1, C_2, \ldots, C_k$ of M where C_1 is start configuration of M on input w, each C_i yields C_{i+1} and C_k is accepting configuration}

"The language of M"

"The language recognized by M"

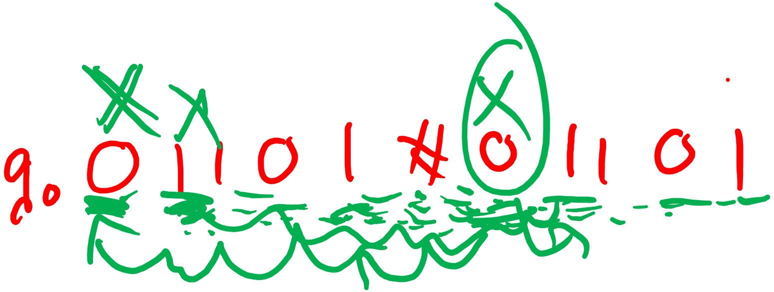
An example

$\{ w\#w^R \mid w \text{ is in } \{0,1\}^* \}$ is context-free.

L = { w#w | w is in $\{0,1\}^*$ }

We already know that L is

- not regular
- not context-free



We will prove that L is

the language of some Turing machine

Turing-recognizable.

Implementation-level description

$$L = \{ w\#w \mid w \text{ is in } \{0,1\}^* \}$$

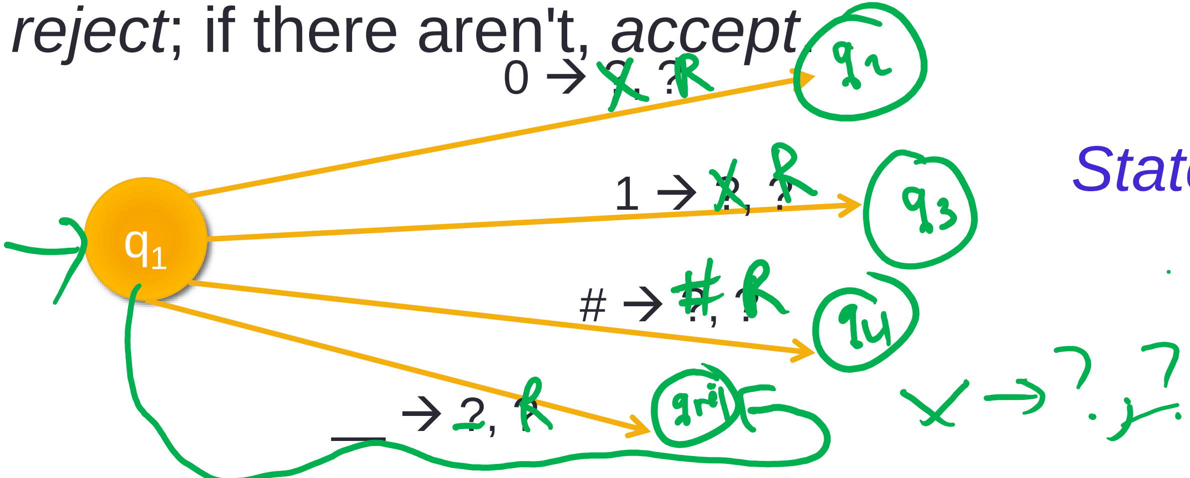
Idea for Turing machine

- Zig-zag across tape to corresponding positions on either side of '#' to check whether these positions agree. If they do not, or if there is no '#', *reject*. If they do, cross them off.
- Once all symbols to the left of the '#' are crossed off, check for any un-crossed-off symbols to the right of '#': if there are any, *reject*; if there aren't, *accept*.

Implementation-level description

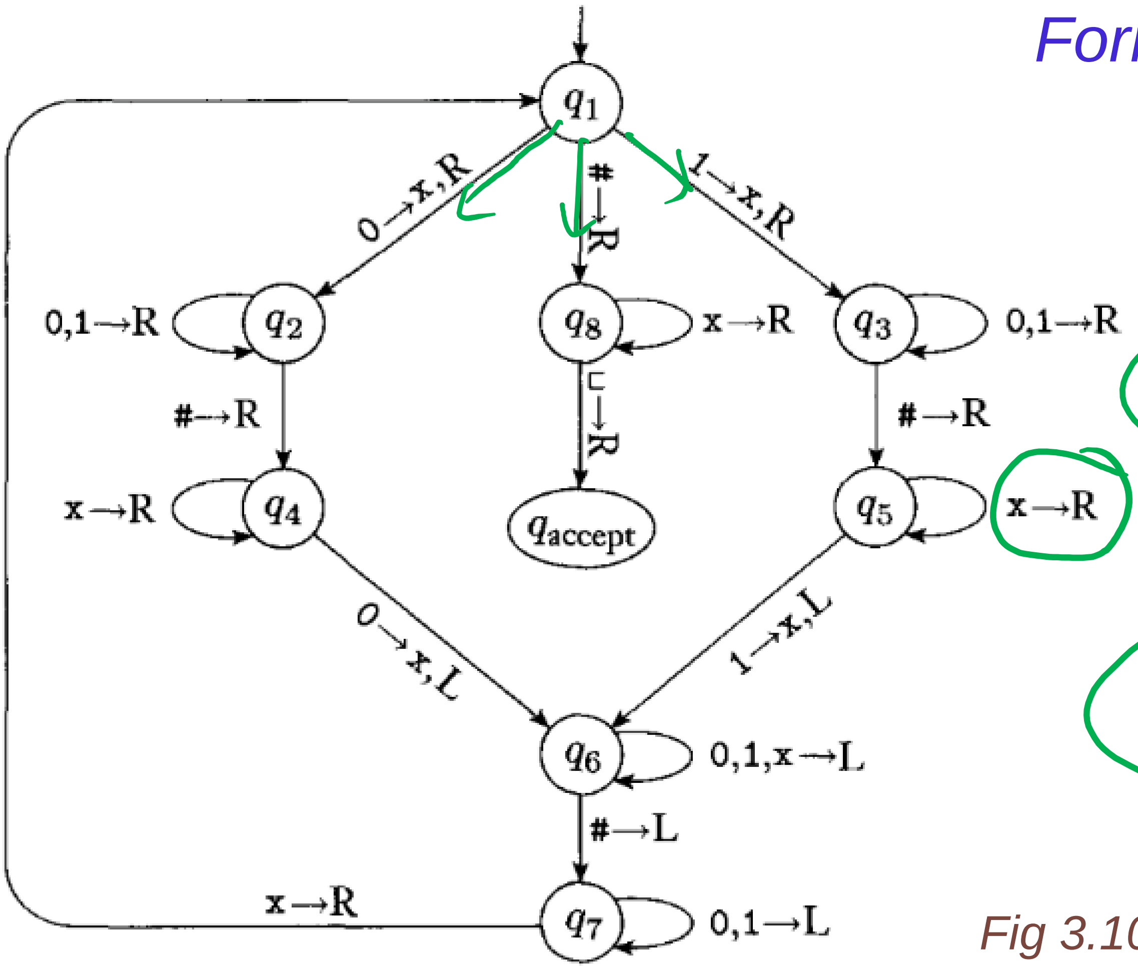
Zig-zag across tape to corresponding positions on either side of '#' to check whether these positions agree. If they do not, or if there is no '#', *reject*. If they do, cross them off.

Once all symbols to the left of the '#' are crossed off, check for any un-crossed-off symbols to the right of '#': if there are any, *reject*; if there aren't, *accept*.



State diagram

Formal definition



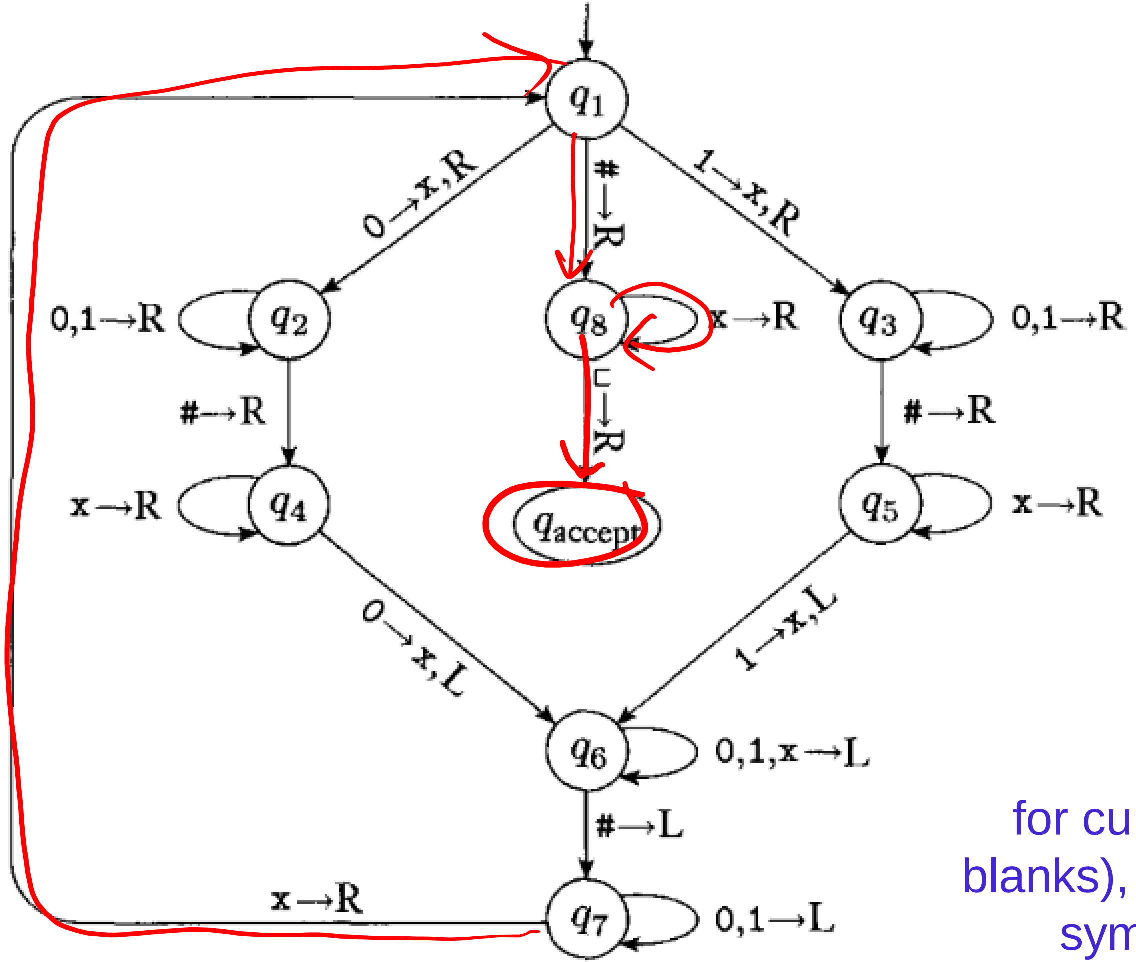
$Q=\{q1,q2,q3,q4,q5,q6,q7,q8,q_{accept},q_{reject}\}$

$\Sigma = \{0,1,\#\}$

$\Gamma = \{0,1,\#,x,\_\}$

All missing transitions have output (q_{reject}, _, R)

Fig 3.10 in Sipser

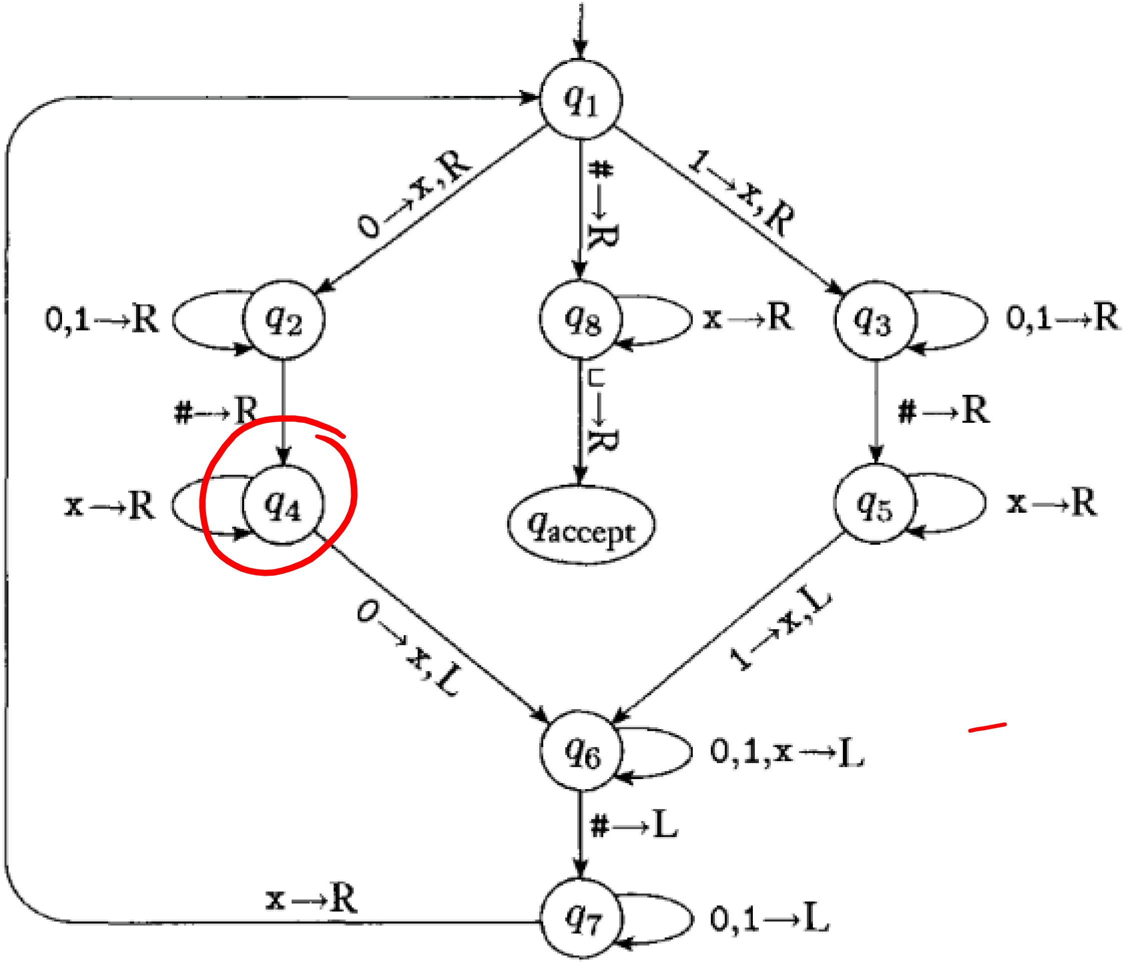


Computation on input 0#0?

q_1 0 # 0
x q_2 # 0
x # q_4 0
x q_6 # x
q_7 x # x
x q_1 # x
x # q_8 x

x # x q_8
x # x ⊔ q_{acc}

Configuration **u q v**
for current tape uv (and then all blanks), current head location is first symbol of v, current state q



Computation on input 0# ?

q_1 0 #

x q_2 #

x # q_4 ␣

x # ␣ q_{rej}

For next time

GroupHW4 due Saturday, February 17

For Friday, pre-class reading: pp. 184-185