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**INSTRUCTIONS**

This **Individual HW8** covers the material from Chapter 7 you should be comfortable with. You can use it to help you study for the final exam.

READING Sipser Highlights from Chapter 7.

KEY CONCEPTS Running time of a Turing machine, time complexity classes, polynomial time ( $P$ ), proving that a problem is in  $P$ ,  $NP$ , proving that a problem is in  $NP$ , polynomial-time reduction,  $NP$ -completeness, examples of  $NP$ -complete problems, using reduction to prove  $NP$ -completeness

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1.

a. Prove that  $P$  is closed under complementation.

b. It is not currently known whether  $NP$  is closed under complementation. Would knowing that (i)  $NP$  is closed under complementation or that (ii)  $NP$  is not closed under complementation give us any information about whether  $P = NP$ ? Briefly justify your answer.

2. Let  $n$  be the number of states in a DFA. Show that  $E_{DFA} \in P$  by showing that the algorithm in the proof of Theorem 4.4 (that  $E_{DFA}$  is a decidable language) runs in polynomial time (as a function of  $n$ ).

3. In class (on Wed, March 14, Pi day) we gave a polynomial-time reduction from 3SAT to CLIQUE. You can also find the details on page 302 of the textbook. Apply this reduction to the formula

$$\phi = (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee \bar{x}_2) \wedge (\bar{x}_2 \vee \bar{x}_3 \vee x_3)$$

with 4 clauses. (We are using the book's convention that  $\bar{x}$  means  $\neg x$ ). Is the formula satisfiable? Does the graph have a 4-clique? *Hint: the answers to these questions should both be yes or both be no.*