CSE 105
Individual Homework 7
Due: Tuesday March 13, 2018 at 11:00PM (on Gradescope)

INSTRUCTIONS
This **Individual HW7** must be completed without any collaboration with other students in this class. The only allowed sources of help for this homework are the class textbook, notes, and podcast, and the instructional team. Two of the questions on this homework will be graded for fair effort completeness; one will be graded for correctness. Your homework **must be typed**.

READING Sipser Sections 4.2, 5.1

KEY CONCEPTS Undecidable problems, recognizable and co-recognizable problems, reductions.

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1. (**10 points**) **True/False** Briefly justify each answer:

a. The class of recognizable languages is closed under complementation.

b. If a language is undecidable then it is infinite.

c. If a language A is regular and B reduces to A then B is also regular.

d. Suppose A is recognizable. If A reduces to B and $\overline{B}$ reduces to A then B is recognizable.

Recall reduction is defined in section 5.1 (it is **not the same** as the m-reduction of later sections).

2. (**10 points**)

a. Prove that A_{TM} reduces to $\overline{A_{TM}}$ (its complement).

b. Prove that $\overline{A_{TM}}$ reduces to A_{TM}.

3. Consider the following computational problems

$$Z_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } 0 \in L(M)\}$$
$$T2_{TM} = \{\langle M \rangle \mid M \text{ is a Turing machine and } |L(M)| \geq 2\}$$

Complete the proof that Z_{TM} reduces to $T2_{TM}$ by filling in the appropriate blanks.
Proof: We will use access to a genie G for $T2_{TM}$ in order to define a genie-decider for Z_{TM}. Define

M_Z = "On input $\langle M \rangle$:

0. Check that input is valid encoding of a Turing machine. If not, reject.
1. Build a new TM X (over the alphabet $\{0, 1\}$) defined as follows :

   X = "On input x:

   1. If x has length greater than 1, reject.
   2. If $x = \varepsilon$, reject.
   3. If $x = 0$, accept.
   4. Otherwise, simulate M on 0.
      If this simulation accepts, ____________________; If it rejects,____________________."

2. Ask the genie G about input __________________.
3. If the genie accepts, ____________________; If it rejects,____________________."

The key observations in the correctness proof of this construction, are that for any TM M,

- if $\langle M \rangle \in Z_{TM}$, then $L(X)$ equals ____________, and M_Z accepts $\langle M \rangle$;
- if $\langle M \rangle \notin Z_{TM}$, then $L(X)$ equals ____________, and M_Z rejects $\langle M \rangle$.

Thus, $L(M_Z) = Z_{TM}$.