CSE 105
Individual Homework 3
Due: Tuesday January 30, 2018 at 11:00PM (on Gradescope)

Instructions
This **Individual HW3** must be completed without any collaboration with other students in this class. The only allowed sources of help for this homework are the class textbook, notes, and podcast, and the instructional team. Two of the questions on this homework will be graded for fair effort completeness; one will be graded for correctness. Your homework **must be typed**.

Reading Sipser Section 1.2, 1.3, 1.4

Key Concepts Regular languages, closure of the class of regular languages under certain operations, nondeterministic finite automata (NFA), spontaneous transitions (ε arrows), equivalence of NFA and DFA and regular expressions, nonregular languages, pumping lemma.

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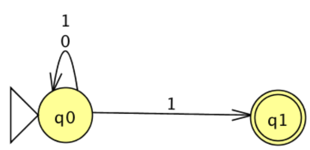
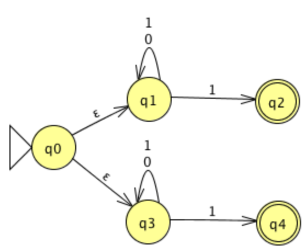
1. (**10 points**) Consider the NFA M:

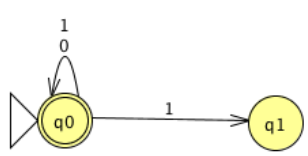


(a) According to Theorem 1.45, the NFA, N, that would result from the general algorithm for proving that $L(M) \cup L(M)$ is regular is given by the state diagram:



- Draw the computation of N on the input 001 (it should be a tree of runs, like that on page 49).
- Explain why $L(N) = L(M)$.

(b) For a DFA, switching accept / non-accept states gives a machine that recognizes the complement of the language of the original machine. Does



recognize $\overline{L(M)}$? Why or why not?

(c) Compute a regular expression describing $L(M)$. You can use your earlier work or use the Theorems in the book to convert M to a DFA and then to a regular expression. Include enough work to justify your answer.

2. Show that the class of regular languages over the alphabet $\{0, 1\}$ is closed under the operation $Reverse(L)$, defined as

$$Reverse(L) = \{w \mid w^{\mathcal{R}} \in L\}$$

A full proof would have three stages: setup, construction, and proof of correctness. In this exercise you will focus on the setup and construction, and then apply your construction to an example.

Setup Consider an arbitrary NFA $N = (Q, \{0, 1\}, \delta, q_0, F)$, and call the language of this NFA L.

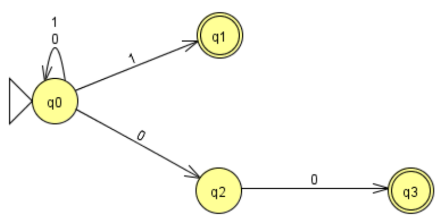
Construction Build a new NFA whose language is $Reverse(L)$. To do so, fill in the blanks

$$N' = (Q', \{0, 1\}, \delta', q', F')$$

where

$Q' =$	__________________	This will be the set of states for your new machine.
$\delta'((r, x)) =$	__________________	For each possible input to the transition function, specify the output. Notice that r is a state in Q' and $x \in \{0, 1, \varepsilon\}$.
$q' =$	__________________	What is the initial state of N'? Make sure you choose an element of Q'.
$F' =$	__________________	What is the set of accepting states of N'? Choose a subset of Q'.

Application Consider the language, L, recognized by this NFA:



Apply your construction to this NFA and confirm that the language recognized by the resulting NFA is $Reverse(L)$.

[[*Bonus (not for credit)*: To prove that the construction of correct, we would need to prove that $L(N') = Reverse(L)$ for any L. Fix an arbitrary but unknown language L. Let N be a NFA recognizing L, and construct N' from N as shown above. Two claims are required

(1) Assume that some string, call it w, is accepted by N'. Prove that w is in $Reverse(L)$.

(2) Assume that some string, call it y, is in $Reverse(L)$. Prove that y is accepted by N'.

Practice your proof techniques by carrying out this justification.]]

3. Consider the language $L = \{uw \mid u \text{ and } w \text{ are strings over } \{0,1\} \text{ and have the same number of 1s}\}$. Find the error in the following attempted proof that L is not regular:

> "Proof" that L is not regular using the Pumping Lemma: Assume (towards a contradiction) that L is regular. Then the Pumping Lemma applies to L. Let p be the pumping length of L. Choose s to be the string $1^p0^p1^p0^p$, which is in L because we can choose $u = 1^p0^p$ and $w = 1^p0^p$ which each have p 1s.
>
> Since s is in L and has length greater than or equal to p, the Pumping Lemma guarantees that s can be divided into parts x, y, z where $s = xyz$, $|y| > 0$, $|xy| \leq p$, and for each $i \geq 0$, $xy^iz \in L$. Since the first p letters of s are all 1 and $|xy| \leq p$, we know that x and y are made up of all 1s. If we let $i = 2$, we get a string xy^iz that is not in L because repeating y twice adds 1s to u but not to w, and strings in L are required to have the same number of 1s in u as in w.
>
> This is a contradiction. Therefore, the assumption is false, and L is not regular.