
Instructions

This **Individual HW2** must be completed without any collaboration with other students in this class. The only allowed sources of help for this homework are the class textbook, notes, and podcast, and the instructional team. Two of the questions on this homework will be graded for fair effort completeness; one will be graded for correctness.

Your homework **must be typed**. We recommend that you submit early drafts to Gradescope so that in case of any technical difficulties, at least some of your work is present. You may update your submission as many times as you'd like up to the deadline.

Your assignments in this class will be evaluated not only on the correctness of your answers, but also on your ability to present your ideas clearly and logically. Whether you use formal proof techniques or write a more informal argument for why something is true, **your answers should always be well-supported**.

Reading Sipser Section 1.1, 1.2

Key Concepts Regular languages, closure of the class of regular languages under certain operations, nondeterministic finite automata (NFA), nondeterministic computation, spontaneous transitions (ε arrows).

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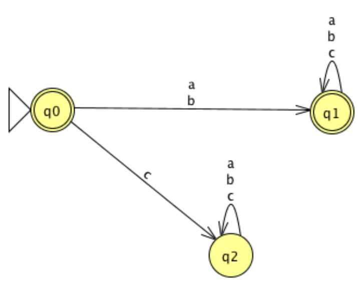
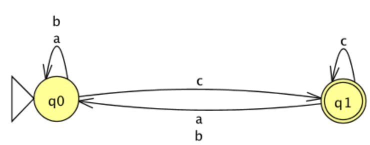
1. (**10 points**) Consider the DFAs M_1 (on the left) and M_2 (on the right):





(a) If we use the general constructions discussed in class and in the book for building a DFA whose language is

$$\overline{L(M_1)} \cup L(M_2)$$

how many states would be in this DFA? Briefly justify your calculation.

(b) Draw the state diagram that results from this construction and remove any unreachable states. How many states are left?

(c) Describe the language $\overline{L(M_1)} \cup L(M_2)$ in set builder notation. You may find it useful to first describe $L(M_1)$ and $L(M_2)$ in set builder notation first.

(d) Use the description from (c) to draw a DFA with fewer states even than what you saw in part (b). Draw the state diagram in JFLAP and include the image in your submission.

Note: For parts (b) and (d), you can use the "test equivalence" feature of JFLAP to check your work.

2. Show that the class of regular languages over the alphabet $\{0,1\}$ is closed under the operation $FlipBits(L)$, defined as

$$FlipBits(L) = \{w \mid w \text{ can be obtained from some } w' \text{ in } L \text{ by flipping each 0 in } w' \text{ to 1 and each 1 to 0}\}$$

A full proof would have three stages: setup, construction, and proof of correctness. In this exercise you will focus on the setup and construction, and then apply your construction to an example.

Setup Consider an arbitrary DFA $M = (Q, \{0,1\}, \delta, q_0, F)$, and call the language of this DFA L.

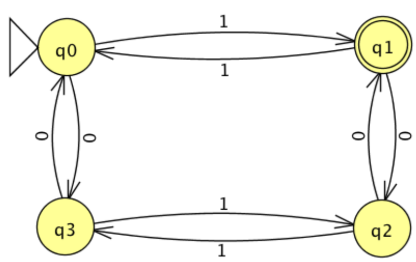
Construction Build a new DFA whose language is $FlipBits(L)$. To do so, fill in the blanks

$$M' = (Q', \{0,1\}, \delta', q', F')$$

where

$Q' =$ ________________	This will be the set of states for your new machine.
$\delta'((r,x)) =$ ________________	For each possible input to the transition function, specify the output. Notice that r is a state in Q' and $x \in \{0,1\}$.
$q' =$ ________________	What is the initial state of M'? Make sure you choose an element of Q'.
$F' =$ ________________	What is the set of accepting states of M'? Choose a subset of Q'.

Application Consider the language, L, recognized by this DFA (from HW1):



Apply your construction to this DFA and confirm that the language recognized by the resulting DFA is $FlipBits(L)$.

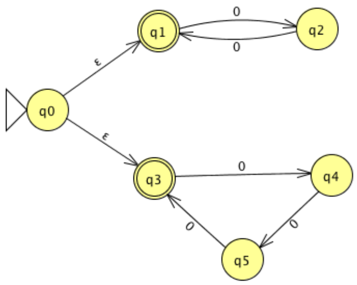
[[*Bonus (not for credit)*: To prove that the construction of correct, we would need to prove that $L(M') = FlipBits(L)$ for any L. Fix an arbitrary but unknown language L. Let M be a DFA recognizing L, and construct M' from M as shown above. Two claims are required

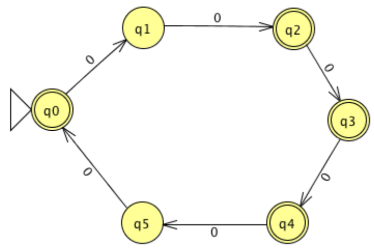
(1) Assume that some string, call it w, is accepted by M'. Prove that w is in $FlipBits(L)$.

(2) Assume that some string, call it y, is in $FlipBits(L)$. Prove that y is accepted by M'.

Practice your proof techniques by carrying out this justification.]]

3. Consider the NFAs N_1 (on the left) and N_2 (on the right):





(N_1 is described in example 1.33 on page 52.)

(a) Write out the formal definition of N_1 and N_2. *Notice that N_2 appears to be a DFA but we are asking for its formal description as a NFA. Be careful of types, especially when describing the transition function of each NFA.*

(b) Write out $L(N_1)$ and $L(N_2)$ in set builder notation, justifying each briefly by making specific references to the state diagrams.

(c) In fact, $L(N_1) = L(N_2)$. You can check this by using the "test equivalence" feature of JFLAP to compare the two state diagrams. Explain why these two NFAs are equivalent.