CSE 20
DISCRETE MATH

Winter 2017

http://cseweb.ucsd.edu/classes/wi17/cse20-ab/

Reminders

- Exam 1 on Tuesday January 31
  - One note card can be used. Bring photo ID.
  - Review sessions
    - Today!
    - Saturday January 28 11am-12:50pm PETER108 Recommended textbook qs
    - Monday January 30 8pm-9:50pm SOLIS107 Exam Practice Sheet
  - Assigned seats: seat map on Piazza
- Office hours listed on class calendar
- HW 3 due Sunday at noon

Reviewing learning goals

- Trace pseudocode given input.
- Explain the higher-level function of an algorithm expressed with pseudocode.
- Identify and explain (informally) whether and why given pseudocode satisfies properties of being an algorithm.
- Give counterexamples to show how an algorithm fails to be correct.
- Define the greedy approach for an optimization problem and analyze whether the greedy approach solves an optimization problem.
- Define the decimal, binary, hexadecimal, and octal expansions of a positive integer and convert between expansions in different bases of a positive integer.
- Define and use the DIV and MOD operators.
- Describe and use algorithms for integer operations based on their expansions and relate algorithms for integer operations to bitwise boolean operations
- Translate sentences from English to propositional logic using appropriate propositional variables and boolean operators.
- Recognize and use truth tables: negation, conjunction, disjunction, exclusive or, conditional, biconditional operators.
- Evaluate the truth value of a compound proposition given truth values of its constituent variables.
- Form the converse, contrapositive, and inverse of a given conditional statement.
- Decide and justify whether or not a collection of propositions is consistent.
- Identify when and prove that a statement is a tautology or contradiction
- Prove propositional equivalences using truth tables
- Prove propositional equivalences using other known equivalences, e.g.
  - DeMorgan's laws
  - Double negation laws
  - Distributive laws, etc.
- Compute the CNF and DNF of a given compound proposition.
- Relate boolean operations to applications

Combinatorial circuits

Logic puzzles

How do we decide (and prove) what's true?
How do we use properties to build systems?
What's impossible?

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- Compute the CNF and DNF of a given compound proposition.
- Relate boolean operations to applications
  - Combinatorial circuits
  - Logic puzzles

Implement a proposition

What combinatorial logic circuit (with AND, OR, NOT, XOR gates) implements the compound proposition

$$(p \to q) \leftrightarrow (r \to p) \quad ?$$

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Strategy: Find equivalent proposition and then implement.

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What combinatorial logic circuit (with AND, OR, NOT, XOR gates) implements the compound proposition

$$(p \rightarrow q) \leftrightarrow (r \rightarrow p) \quad ?$$

Strategy: Find equivalent proposition and then implement.

- Via logical equivalences
- Via truth table algorithm for CNF / DNF

Via logical equivalences

$$(p \rightarrow q) \leftrightarrow (r \rightarrow p)$$

$$\equiv \neg((p \rightarrow q) \oplus (r \rightarrow p))$$

$$\equiv \neg((\neg p \vee q) \oplus (\neg r \vee p))$$

Via truth table

p	q	r	$(p \to q)$	$(r \to p)$	$(p \to q) \leftrightarrow (r \to p)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Via truth table

p	q	r	$(p \to q)$	$(r \to p)$	$(p \to q) \leftrightarrow (r \to p)$
T	T	T	T		
T	T	F	T		
T	F	T	F		
T	F	F	F		
F	T	T	T		
F	T	F	T		
F	F	T	T		
F	F	F	T		

Via truth table

p	q	r	$(p \to q)$	$(r \to p)$	$(p \to q) \leftrightarrow (r \to p)$
T	T	T	T	T	
T	T	F	T	T	
T	F	T	F	T	
T	F	F	F	T	
F	T	T	T	F	
F	T	F	T	T	
F	F	T	T	F	
F	F	F	T	T	

Via truth table

p	q	r	$(p \to q)$	$(r \to p)$	$(p \to q) \leftrightarrow (r \to p)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	F	F
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

Via truth table

p	q	r	$(p \to q) \leftrightarrow (r \to p)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

LAND IN THESE ROWS!

Via truth table

p	q	r	$(p \to q) \leftrightarrow (r \to p)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

What compound proposition describes the first row?

A. $p \vee q \vee r$
B. $p \wedge q \wedge r$
C. $\neg p \vee \neg q \vee \neg r$
D. $\neg p \wedge \neg q \wedge \neg r$
E. None of the above

Clicker frequency: AC

Via truth table

p	q	r	$(p \to q) \leftrightarrow (r \to p)$	
T	T	T	T	$p \wedge q \wedge r$
T	T	F	T	$p \wedge q \wedge \neg r$
T	F	T	F	
T	F	F	F	
F	T	T	F	
F	T	F	T	$\neg p \wedge q \wedge \neg r$
F	F	T	F	
F	F	F	T	$\neg p \wedge \neg q \wedge \neg r$

Via truth table

p	q	r	$(p \to q) \leftrightarrow (r \to p)$	
T	T	T	T	$p \wedge q \wedge r$
T	T	F	T	$p \wedge q \wedge \neg r$
T	F	T	F	
T	F	F	F	
F	T	T	F	
F	T	F	T	$\neg p \wedge q \wedge \neg r$
F	F	T	F	
F	F	F	T	$\neg p \wedge \neg q \wedge \neg r$

DNF: $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$

As a circuit (assume 3 and 4 input gates are available; otherwise cascade)

DNF: $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge \neg r)$

Via truth table

p	q	r	$(p \to q) \leftrightarrow (r \to p)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

AVOID THESE ROWS!

Via truth table

p	q	r	$(p \rightarrow q) \leftrightarrow (r \rightarrow p)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

What compound proposition describes avoiding the third row?

A. $p \wedge \neg q \wedge r$
B. $\neg(p \wedge \neg q \wedge r)$
C. $p \vee \neg q \vee r$
D. $\neg(p \vee \neg q \vee r)$
E. None of the above

Via truth table

p	q	r	$(p \to q) \leftrightarrow (r \to p)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

$\neg(p \wedge \neg q \wedge r) \equiv \neg p \vee q \vee \neg r$

$\neg(p \wedge \neg q \wedge \neg r) \equiv \neg p \vee q \vee r$

$\neg(\neg p \wedge q \wedge r) \equiv p \vee \neg q \vee \neg r$

$\neg(\neg p \wedge \neg q \wedge r) \equiv p \vee q \vee \neg r$

Via truth table

p	q	r	$(p \to q) \leftrightarrow (r \to p)$	
T	T	T	T	
T	T	F	T	
T	F	T	F ←	$\neg(p \wedge \neg q \wedge r) \equiv \neg p \vee q \vee \neg r$
T	F	F	F ←	$\neg(p \wedge \neg q \wedge \neg r) \equiv \neg p \vee q \vee r$
F	T	T	F ←	$\neg(\neg p \wedge q \wedge r) \equiv p \vee \neg q \vee \neg r$
F	T	F	T	
F	F	T	F ←	$\neg(\neg p \wedge \neg q \wedge r) \equiv p \vee q \vee \neg r$
F	F	F	T	

CNF: $(\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee q \vee \neg r)$

As a circuit (assume 3 and 4 input gates are available; otherwise cascade)

CNF: $(\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee q \vee \neg r)$

Puzzle: Muddy children

Rosen Example 8, page 20

Two kids play in the mud outside. When they are done, their parent says "at least one of you has a muddy forehead." The children can't see themselves but can see each other. If the older child sees mud on the younger child's forehead …

A. The older child knows both of them have muddy foreheads.
B. The older child knows the younger child has a muddy forehead.
C. The older child knows that they themselves have a muddy forehead.
D. None of the above – they can't be sure.
E. I don't know.

Puzzle: Muddy children

Rosen Example 8, page 20

Two kids play in the mud outside. When they are done, their parent says “at least one of you has a muddy forehead.” The children can’t see themselves but can see each other. If the older child (even being a perfect reasoner) doesn't have enough information to tell whether s/he has mud on their forehead…

A. The younger child knows the older one has muddy forehead.
B. The younger child knows the older one has no mud.
C. The younger child knows s/he himself has no mud.
D. None of the above – they can’t be sure.
E. I don't know.

Translations

If 2 is an integer then so is 3.

If 2 is an integer then so is 0.4.

If 2 is not an integer, then neither is 0.5.

If 2 is not an integer, then neither is 33.

Envelopes for binary expansion

You have one thousand $1 bills. How can you distribute them among ten envelopes so that any amount between $1 and $1000, inclusive, can be given as some combination of these envelopes? No change is allowed, and you are not allowed to open any of the envelopes once you've determined how many bills to put in each at the start.

Envelopes strategy

$247?

$980?

Exam strategy

- 5 questions: topics as indicated on practice exam.
- Questions are listed by topic, not by difficulty.
- **Read all questions.**
- **Start with ones you know how to do.**
- Read directions carefully. (Need to justify? What's required?)
- Ask questions if needed.