CSE 20
DISCRETE MATH

Winter 2017

http://cseweb.ucsd.edu/classes/wi17/cse20-ab/

Today's learning goals

- Translate sentences from English to propositional logic using appropriate propositional variables and boolean operators.
- Truth tables: negation, conjunction, disjunction, exclusive or, conditional, biconditional operators.
- Evaluate the truth value of a compound proposition given truth values of its constituent variables.
- Form the converse, contrapositive, and inverse of a given conditional statement.
- Decide and justify whether or not a collection of propositions is consistent.

Definitions

Rosen p. 2-4

- **Proposition**: declarative sentence that is T or F (not both)
- **Propositional variable**: variables that represent propositions.
- **Compound proposition**: new propositions formed from existing propositions using logical operators.
- **Truth table**: table with 1 row for each of the possible combinations of truth values of the input and an additional column that shows the truth value of the result of the operation corresponding to a particular row.

Propositions

CSE 20 Remote frequency: **AC**

To change your remote frequency

1. Press and hold power button until flashing
2. Enter two-letter code
3. Checkmark / green light indicates success

Which of the following is a proposition?

A. Answer this question.
B. What time is it?
C. $4 + x = 5$.
D. $2^3 > 8$.
E. None of the above.

Truth tables

Can use truth table to compute value of compound proposition.

p	q	r	(p v q) v (p v r)
1	1	1	
1	1	0	
1	0	1	
1	0	0	
0	1	1	
0	1	0	
0	0	1	
0	0	0	

Truth tables

Can use truth table to compute value of compound proposition.

p	q	r	p v q	p v r	(p v q) v (p v r)
1	1	1			
1	1	0			
1	0	1			
1	0	0			
0	1	1			
0	1	0			
0	0	1			
0	0	0			

Circuit?

Compound propositions

Rosen p. 3-4

p	¬ p
T	F
F	T

p	q	p ∨ q p OR q	p ∧ q p AND q	p ⊕ q p XOR q
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

"p OR q is T if at least one of p or q is T"

"p AND q is T if both p and q are T"

"p XOR q is T if exactly one of p and q is T"

Compound propositions

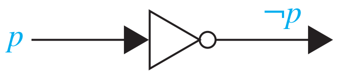
Rosen p. 3-4, 21

p	¬ p
T	F
F	T

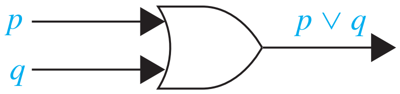
Negation

p	q	p ∨ q p OR q	p ∧ q p AND q	p ⊕ q p XOR q
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

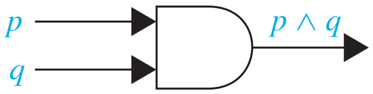
Disjunction Conjunction



Inverter



OR gate



AND gate

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg(\neg p \vee \neg q)$
T	T	?
T	F	?
F	T	?
F	F	?

To fill in rows

Plug in values one row at a time.

OR

Use intermediate columns.

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
T	T				?
T	F				?
F	T				?
F	F				?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
T	T	**F**			?
T	F	**F**			?
F	T	**T**			?
F	F	**T**			?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
T	T	F	**F**		?
T	F	F	**T**		?
F	T	T	**F**		?
F	F	T	**T**		?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \lor \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \lor \neg q$	$\neg(\neg p \lor \neg q)$
T	T	F	F	**F**	?
T	F	F	T	**T**	?
F	T	T	F	**T**	?
F	F	T	T	**T**	?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \lor \neg q)$$

p	**q**	$\neg p$	$\neg q$	$\neg p \lor \neg q$	$\neg(\neg p \lor \neg q)$
T	T	F	F	F	**T**
T	F	F	T	T	**F**
F	T	T	F	T	**F**
F	F	T	T	T	**F**

Does this look familiar?

Logical equivalences

Rosen p. 25

Compound propositions that have the same truth values in all possible cases are **logically equivalent**, denoted $\equiv$.

p	q	$\neg(\neg p \vee \neg q)$
T	T	T
T	F	F
F	T	F
F	F	F

What compound proposition is logically equivalent to $\neg(\neg p \vee \neg q)$?

A. $p \wedge q$
B. $p \vee q$
C. $p \wedge \neg p$
D. $q \vee \neg q$
E. None of the above.

Translation

Rosen p. 22: 1.2#7

Express the sentence
"The message was sent from an unknown system but it was not scanned for viruses" using the propositions

p: "The message is scanned for viruses"
q: "The message was sent from an unknown system"

A. $p \wedge q$
B. $p \wedge \neg q$
C. $\neg p \vee q$
D. $p \vee \neg q$
E. None of the above.

Conditionals

Rosen p. 6-10

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

"If p, then q"

Conditionals

Rosen p. 6-10

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

"If p, then q"

The only way to make a conditional statement false is to …

Conditionals

Rosen p. 6-10

Hypothesis
Antecedent

Conclusion
Consequent

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

"If p, then q"

Conditionals

Rosen p. 6-10

Which of these compound propositions **is not** logically equivalent to p → q ?

A. $\neg p \vee q$

B. $\neg(p \wedge \neg q)$

C. $q \rightarrow p$

D. $\neg q \rightarrow \neg p$

E. None of the above.

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Conditionals

Rosen p. 6-10

Which of these compound propositions
is not logically equivalent to p → q ?

A. $\neg p \vee q$

B. $\neg(p \wedge \neg q)$

C. $q \rightarrow p$

D. $\neg q \rightarrow \neg p$

E. None of the above.

Circuits?

Conditionals

Rosen p. 6-10

p	q	p → q	q → p	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	T			
T	F	F			
F	T	T			
F	F	T			

Converse of p→q

Contrapositive of p→ q

Inverse of p→q

Conditionals

Rosen p. 6-10

p	q	p → q	q → p	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	T			
T	F	F		F	
F	T	T	F		F
F	F	T			

Converse of p→q

Contrapositive of p→ q

Inverse of p→q

Conditionals

Rosen p. 6-10

p	q	p → q	q → p	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Converse of p→q

Contrapositive of p→ q

Inverse of p→q

Biconditionals

Rosen p. 6-10

Which of these compound propositions **is** logically equivalent to $p \leftrightarrow q$?

A. $p \rightarrow q$

B. $p \wedge q$

C. $p \vee q$

D. $p \oplus q$

E. None of the above.

"If and only if"
"Necessary and sufficient"

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Translation

Rosen p. 22: 1.2#7

Express the sentence
"The message is scanned for viruses whenever the message was sent from an unknown system" using the propositions

p: "The message is scanned for viruses"
q: "The message was sent from an unknown system"

A. $p \wedge q$
B. $p \vee q$
C. $p \rightarrow q$
D. $p \leftrightarrow q$
E. None of the above.

System specification + consistency

Rosen p. 23 #11

The router can send packets to the edge system only if it supports the new address space.

For the router to support the new address space, it is necessary that the latest software release be installed.

The router can send packets to the edge system if the latest software release is installed.

The router supports the new address space.

System specification + consistency

Rosen p. 23 #11

p only if q.

For q, it is necessary that r.

p if r.

q.

System specification + consistency

Rosen p. 23 #11

p only if q. $\neg q \rightarrow \neg p$

For q, it is necessary that r. $\neg r \rightarrow \neg q$

p if r. $r \rightarrow p$

q. q

System specification + consistency

Rosen p. 18

p only if q. $\neg q \rightarrow \neg p$

For q, it is necessary that r. $\neg r \rightarrow \neg q$

p if r. $r \rightarrow p$

q. q

Can such a system be built?

System specifications are **consistent** if they do not contain conflicting requirements

In other words: the specifications are consistent if there is a truth assignment to the input propositional variables that makes each specification true.

System specification + consistency

Rosen p. 23 #11

p	q	r	$\neg q \rightarrow \neg p$			
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

How many rows will be set to F in 4th column?

A. 1
B. 2
C. 4
D. 8
E. None of the above

System specification + consistency

Rosen p. 23 #11

p	q	r	$\neg q \to \neg p$	$\neg r \to \neg q$	$r \to p$	q
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	F

System specification + consistency

Rosen p. 18

p	q	r	$\neg q \rightarrow \neg p$	$\neg r \rightarrow \neg q$	$r \rightarrow p$	q
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	F

System specifications are **consistent** if they do not contain conflicting requirements

Tautology and contradiction

Rosen p. 25

Tautology: compound proposition that is always T

Contradiction: compound proposition that is always F

p	q	F	T
T	T	F	T
T	F	F	T
F	T	F	T
F	F	F	T

Which of the following is a tautology?

A. p
B. $p \vee p$
C. $p \wedge p$
D. $p \vee \neg p$
E. $p \wedge \neg p$

Reminders

- Homework 2 due Sunday at noon
  - Bit operations, circuits, logic, propositions.
- Office hours