CSE 20
DISCRETE MATH

There are 10 types of people in the world: those who understand binary and those who don't.

Winter 2017

http://cseweb.ucsd.edu/classes/wi17/cse20-ab/

Today's learning goals

- Define the decimal, binary, hexadecimal, and octal expansions of a positive integer.
- Convert between expansions in different bases of a positive integer.
- Define and use the DIV and MOD operators.

About you

MANDE B-210: AC PETER 108: AC

To change your remote frequency

1. Press and hold power button until flashing
2. Enter two-letter code
3. Checkmark / green light indicates success

How many people in this class have you met so far?

A. None.
B. Less than 5.
C. 5-10.
D. 10-15.
E. More than 15.

Algorithms!

From last time

An **algorithm** is a finite sequence of precise instructions for performing a computation or for solving a problem.

… arithmetic

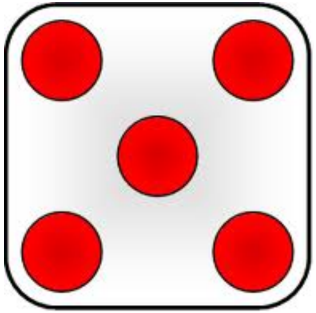
… optimization

Representation

$\log_2(32)$

101

Five

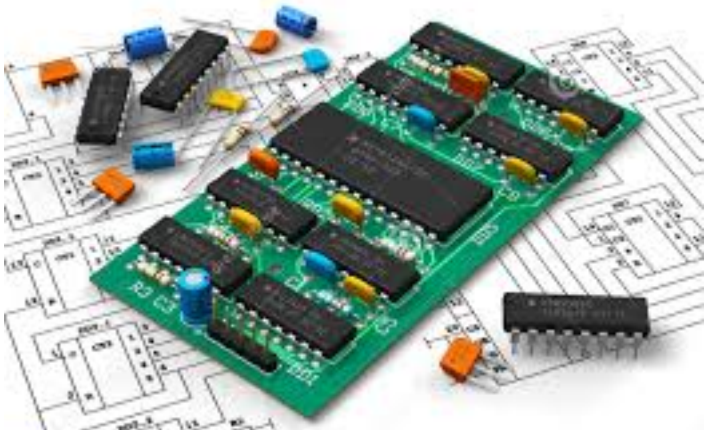


5

Integer representations

Different contexts call for different representations.

Base 10



Base 2

Bases and algorithms

142 x **17**

Decimal expansion: Write numbers as sums of multiples of powers of 10

142 = 1 x 100 + 4 x 10 + 2 x 1

17 = 1 x 10 + 7 x 1

Powers of 10

$142 = 1 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$

$17 = 1 \times 10^1 + 7 \times 10^0$

If n is a positive integer and we express its usual decimal expansion as sums of multiples of power of 10, then

A. The exponents of powers of 10 in the sum are all positive.
B. There is a largest exponent in the sum.
C. A multiple of a missing power of 10 can be added in without changing the value of the sum.
D. More than one of the above.
E. I don't know.

Powers of 10

$$142 = 1 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$$

$$17 = 1 \times 10^1 + 7 \times 10^0$$

If n is a positive integer and we express its usual decimal expansion as sums of multiples of power of 10, then

A. The coefficient of each term must be nonnegative.
B. The coefficient of each term must be less than 10.
C. The same coefficient can appear in more than one term.
D. More than one of the above.
E. I don't know.

Powers of b

Rosen p. 246

For base b (integer greater than 1) and positive integer n there is a unique choice of

- k, a nonnegative integer
- $a_0, a_1, a_2, a_3, \ldots, a_{k-1}$ integers between 0 and b-1, where
- $a_{k-1} \neq 0$ and

$$n = a_{k-1}b^{k-1} + \ldots + a_1 b + a_0$$

Powers of 2

For what positive integer k can we choose

$a_0, a_1, a_2, \ldots, a_{k-1}$ so that $a_k \neq 0$ **and each** a_i **is 0 or 1** with

$$17 = a_{k-1}2^{k-1} + \quad \ldots + a_1 2 + a_0$$

A. k = 17
B. k = 2
C. k = 4
D. k = 5
E. I don't know.

Powers of 2

Find positive integer k and coefficients $a_0, a_1, a_2, \ldots, a_{k-1}$ so that

$$17 = a_{k-1}2^{k-1} + \ldots + a_1 2 + a_0$$

$$= 16 + 1$$

$$= 1*2^4 + 1$$

So k=5 and a_4=1, a_3=0, a_2=0, a_1=0, a_0=1.

Base expansion

Rosen p. 246

Notation: for positive integer n

Write

$$n = (a_{k-1} \ldots a_1 \, a_0)_b$$

when

$$n = a_{k-1}b^{k-1} + \ \ldots + a_1 b + a_0$$

Base b expansion of n

Common bases

Rosen p. 246

- **Decimal expansion** base 10 $(\mathbf{17})_{10}$
- **Binary expansion** base 2 $(\mathbf{10001})_{2}$
- **Octal expansion** base 8 $(\quad)_{8}$
- **Hexadecimal expansion** base 16 $(\quad)_{16}$

Common bases

Rosen p. 246

- **Decimal expansion** base 10 $(17)_{10}$
- **Binary expansion** base 2 $(10001)_2$
- **Octal expansion** base 8 $(\ 21\ )_8$
- **Hexadecimal expansion** base 16 $(\ 11\ )_{16}$

Common bases

Rosen p. 246

- **Decimal expansion** base 10 — $(17)_{10}$
- **Binary expansion** base 2 — $(10001)_2$
- **Octal expansion** base 8 — $(\ 21\ )_8$
- **Hexadecimal expansion** base 16 — $(\ 11\ )_{16}$

What's different?

Hexadecimal coefficients

- 0
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- A
- B
- C
- D
- E
- F

Base expansion

In what base could this expansion be

$$(1401)_?$$

A. Binary (base 2)
B. Octal (base 8)
C. Decimal (base 10)
D. Hexadecimal (base 16)
E. More than one of the above

Base expansion

In what base could this expansion be

$$(1401)_?$$

A. Binary (base 2)
B. Octal (base 8) — Value: $1*8^3+4*8^2+1=768$
C. Decimal (base 10) — Value: $1*10^3+4*10^2+1=1401$
D. Hexadecimal (base 16) — Value: $1*16^3+4*16^2+1=5121$
E. More than one of the above

Algorithm: constructing base b expansion *Rosen p. 249*

Input n,b **Output** k, coefficients in expansion

- English description.

- Pseudocode.

Algorithm: constructing base b expansion *Rosen p. 249*

Input n,b **Output** k, coefficients in expansion

- English description.

  Find k

  Work down to find a_{k-1}, then a_{k-2}, etc.

What's the the base 3 expansion of 17?

A. $(10001)_3$
B. $(210)_3$
C. $(111)_3$
D. $(222)_3$
E. None of the above.

Algorithm: constructing base b expansion *Rosen p. 249*

Input n,b **Output** k, coefficients in expansion

- English description.

  Find k

  Work down to find a_{k-1}, then a_{k-2}, etc.

What's the length of the base b expansion of n?

Start with related question:

What's the smallest power of b that is bigger than n?

Algorithm: constructing base b expansion *Rosen p. 249*

Input n,b **Output** k, coefficients in expansion

- English description.

  Find k by computing successive powers of b until find smallest k such that

  $$b^{k-1} \leq n < b^{k}$$

  For each value of i from 1 to k

  Set a_{k-i} to be the largest number between 0 and b-1 for which $a_{k-i} b^{k-i} \leq n$.

  Update current value remaining $n := n - a_{k-i} b^{k-i}$

Algorithm: constructing base b expansion *Rosen p. 249*

Input n,b **Output** k, coefficients in expansion

- English description.

  Find k by computing successive powers of b until find smallest k such that

  $$b^{k-1} \leq n < b^{k}$$

  For each value of i from 1 to k

  Set a_{k-i} to be the largest number between 0 and b-1 for which $a_{k-i}\, b^{k-l} \leq n$.

  Update current value remaining $n := n - a_{k-i}\, b^{k-i}$

Definite? Finite? Correct?

Challenge: translate to pseudocode!

Arithmetic + Representations *Rosen p. 251*

What is the sum of $(110)_2$ and $(101)_2$?

A. $(011)_2$
B. $(111)_2$
C. $(1011)_2$
D. $(1001)_2$
E. I don't know

Arithmetic + Representations *Rosen p. 251*

What is the product of $(110)_2$ and $(101)_2$?

How can we check our work?
What about in other bases?

Reminders

- Homework 1 due Sunday at noon
  - Set up course tools
  - Pseudocode and algorithms + number representations

- Office hours

There are 10 types of people in the world: those who understand ternary, those who don't, and those who mistake it for binary

(Ternary means base 3)