CSE 20
DISCRETE MATH

Winter 2017

http://cseweb.ucsd.edu/classes/wi17/cse20-ab/

Today's learning goals

- Define and compute the cardinality of a set: Finite sets, countable sets, uncountable sets
- Use functions to compare the sizes of sets
- Determine and prove whether a given binary relation is an equivalence relation using the defining properties
  - symmetry
  - reflexivity
  - transitivity
- Determine and prove whether a given binary relation is an equivalence relation using the associated partition.

Cardinality

Rosen Defn 3 p. 171

- Finite sets — $|A| = n$ for some nonnegative int n
- Countably infinite sets — $|A| = |\mathbf{Z}^{+}|$ (informally, can be listed out)
- Uncountable sets — Infinite but not in bijection with **N**

Cardinality

Rosen Defn 3 p. 171

- Finite sets

|A| = n for some nonnegative int n

Which of the following sets is **not** finite?

A. $\emptyset$

B. $[0, 1]$

C. $\{x \in \mathbb{Z} | x^2 = 1\}$

D. $\mathcal{P}(\{1, 2, 3\})$

E. None of the above (they're all finite)

Cardinality

Rosen p. 172

- Countable sets   A is finite or $|A| = |\mathbf{Z}^+|$ (informally, can be listed out)

Examples: $\emptyset$ $\{x \in \mathbb{Z} | x^2 = 1\}$ $\mathcal{P}(\{1, 2, 3\})$ $\mathbb{Z}^+$

and also …

- the set of **odd positive** integers   Example 1
- the set of **all integers**   Example 3
- the set of **positive rationals**   Example 4
- the set of **negative rationals**
- the set of **rationals**

Cardinality and subsets, redux

Which of the following is **not** true?

A. If A and B are both countable then AUB is countable.
B. If A and B are both countable then A∩B is countable.
C. If A and B are both countable then AxB is countable.
D. If A is countable then P(A) is countable.
E. None of the above

There is an uncountable set! *Rosen example 5, page 173-174*

Cantor's diagonalization argument

Theorem: For every set A, $|A| \neq |\mathcal{P}(A)|$

There is an uncountable set! *Rosen example 5, page 173-174*

Cantor's diagonalization argument

Theorem: For every set A, $|A| \neq |\mathcal{P}(A)|$

An example to see what is necessary. Consider A = {a,b,c}.
What would we need to prove that |A| = |P(A)|?

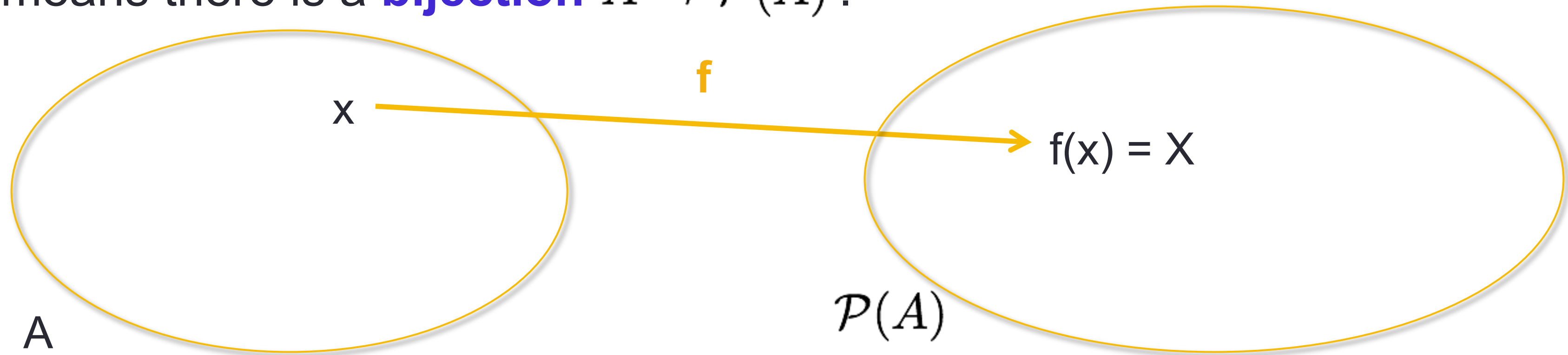
There is an uncountable set! *Rosen example 5, page 173-174*

Cantor's diagonalization argument

Theorem: For every set A, $|A| \neq |\mathcal{P}(A)|$

Proof: (Proof by contradiction)

Assume towards a contradiction that $|A| = |\mathcal{P}(A)|$. By definition, that means there is a **bijection** $A \to \mathcal{P}(A)$.

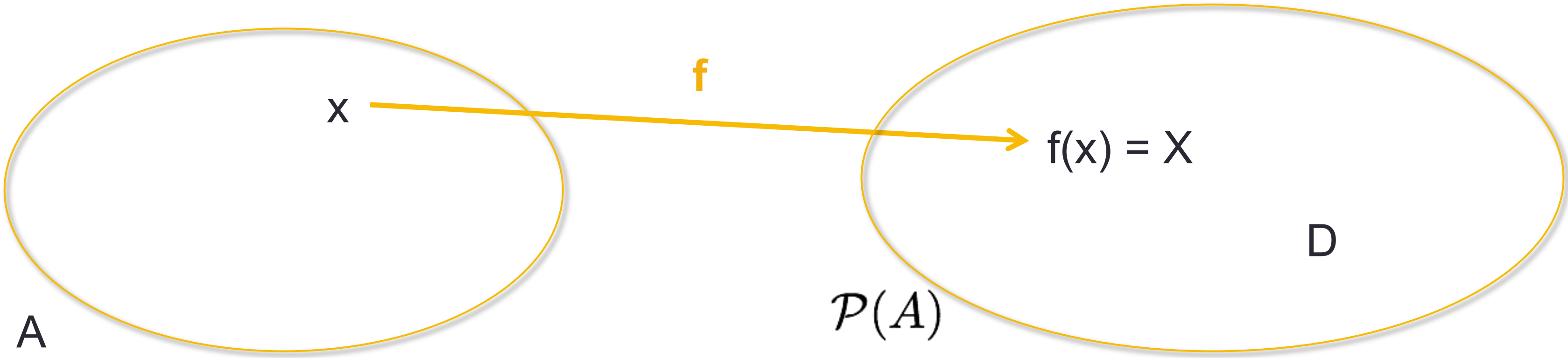


There is an uncountable set! *Rosen example 5, page 173-174*

Cantor's diagonalization argument

Consider the subset D of A defined by, for each a in A:

$$a \in D \qquad \text{iff} \qquad a \notin f(a)$$



There is an uncountable set! *Rosen example 5, page 173-174*

Cantor's diagonalization argument

Consider the subset D of A defined by, for each a in A:

$$a \in D \qquad \text{iff} \qquad a \notin f(a)$$

Define d to be the pre-image of D in A under f   f(d) = D

Is d in D?

- If yes, then by definition of D, $d \notin f(d) = D$ **a contradiction!**
- Else, by definition of D, $\neg(d \notin f(d))$ so $d \in f(D) = D$ **a contradiction!**

Cardinality

Rosen p. 172

- Uncountable sets

Infinite but not in bijection with $\mathbf{Z}^+$

Examples: the power set of any countably infinite set

and also …

- the set of **real** numbers — Example 5
- (0,1) — Example 6 (++)
- (0,1] — Example 6 (++)

Exercises 33, 34

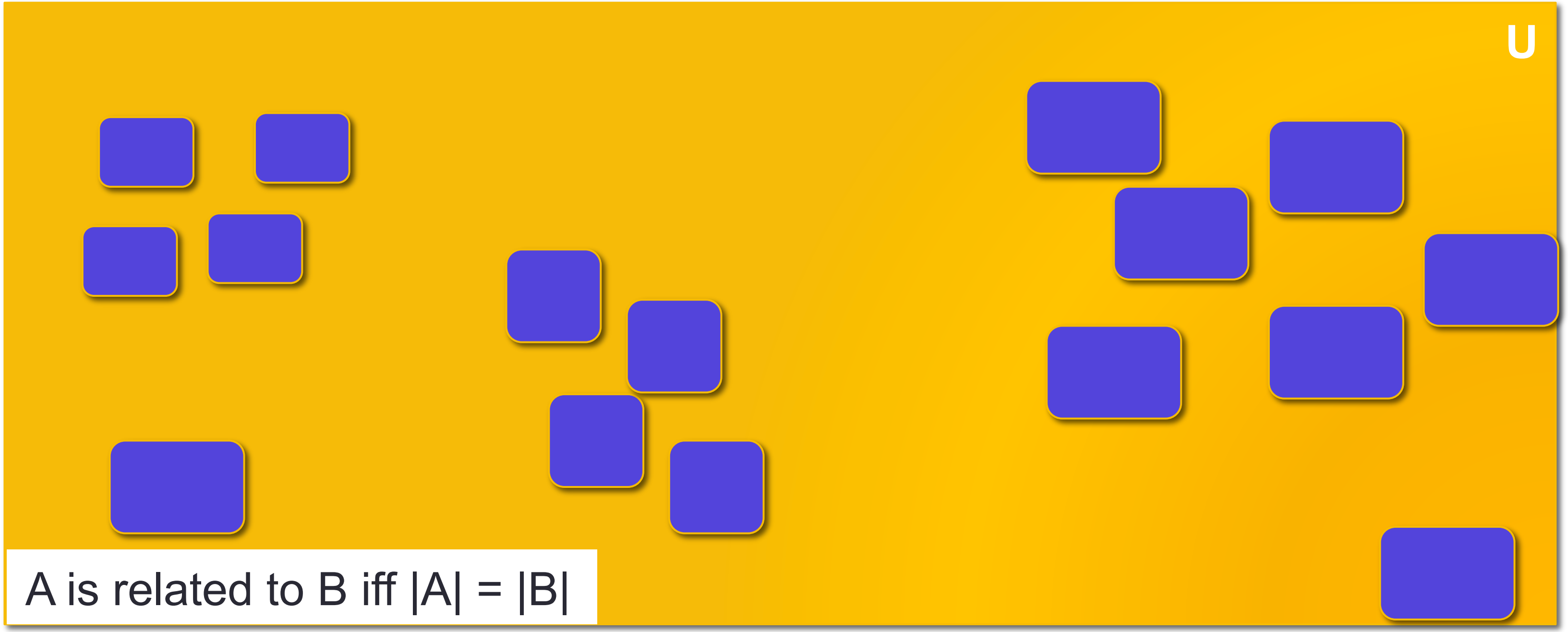
Cardinality and subsets

Suppose A and B are sets and $A \subseteq B$.

A. If A is finite then B is finite.
B. If A is countable then B is uncountable.
C. If B is infinite then A is finite.
D. If B is uncountable then A is uncountable.
E. None of the above.

Size as a relation

- Cardinality lets us compare and group sets.



Size as a relation

- Cardinality lets us compare and group sets.