CSE 20
DISCRETE MATH

Winter 2017

http://cseweb.ucsd.edu/classes/wi17/cse20-ab/

Today's learning goals

- Evaluate which proof technique(s) is appropriate for a given proposition
  - Direct proof
  - Proofs by contraposition
  - Proofs by contradiction
  - Proof by cases
  - Constructive / nonconstructive existence proofs
- Correctly prove statements using appropriate style conventions, guiding text, notation, and terminology
- Define and differentiate between important sets
- Use correct notation when describing sets: {...}, intervals
- Define and prove properties of: subset relation, power set, Cartesian products of sets, union of sets, intersection of sets, disjoint sets, set differences, complement of a set

Overall proof strategy

- Do you believe the statement?
  - Try some small examples.
- Determine logical structure + main connective.
- Determine relevant definitions.
- Map out possible proof strategies.

  Conditional statement? Direct OR contrapositive.
  Existential statement? Find an example.
  Universal statement? Start with generic element …
  Any statement: contradiction

  - For each strategy: what can we **assume**, what is the **goal**?
  - Start with simplest, move to more complicated if/when get stuck.

Some definitions

Rosen Sections 2.1, 2.2

Set: unordered collection of **elements**

A = B iff $\forall x(x \in A \leftrightarrow x \in B)$

How to specify these elements?

- Roster $\{ \ldots \}$
- Set builder $\{x \in U \mid P(x)\}$

Some definitions

Rosen Sections 2.1, 2.2

Set: unordered **collection** of elements

Commas in set builder notation indicate "and"

N: natural numbers {0, 1, 2, 3, …}

Z: integers {…, -2, -1, 0, 1, 2, …}

Z⁺: positive integers {1, 2, 3, …}

Q: rational numbers $\{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, \text{ and } q \neq 0\}$

R: real numbers

R⁺: positive real numbers

C: complex numbers

Some definitions

Rosen Sections 2.1, 2.2

Subset: $A \subseteq B$ means $\forall x(x \in A \rightarrow x \in B)$

Theorem: For any sets A and B, A = B if and only if both $A \subseteq B$ and $B \subseteq A$

Proof:

What's the logical structure of this statement?

A. Universal conditional.

B. Biconditional.

C. Conjunction (and)

D. None of the above.

Some definitions

Rosen Sections 2.1, 2.2

Subset: $A \subseteq B$ means $\forall x (x \in A \rightarrow x \in B)$

Fact: $\mathbf{Z}^+$ is a subset of **Z**

Fact: **Z** is a subset of **Q**

Fact: **Q** is a subset of **R**

Fact: Q is *not* a subset of **Z**

Some definitions

Rosen Sections 2.1, 2.2

Subset: $A \subseteq B$ means $\forall x (x \in A \rightarrow x \in B)$

How would you prove that **R** is not a subset of **Q**?

A. Prove that every real number is not rational.
B. Prove that every rational number is real.
C. Prove that there is a real number that is rational.
D. Prove that there is a real number that is not rational.
E. Prove that there is a rational number that is not real.

An (ir)rational excursion

Rosen p. 86, 97

Theorem: R is not a subset of **Q**.

Lemma: $\sqrt{2}$ is not rational.

Corollary: There are irrational numbers x,y such that x^y is rational.

An (ir)rational excursion

Rosen p. 86, 97

What's the logical structure of each of these statements?

Theorem: R is not a subset of **Q**.

Lemma: √2 is not rational.

Negated existential statement: need a universal argument.

Corollary: There are irrational numbers x,y such that x^y is rational.

Existential statement: can we build a witness?

Proof by contradiction

Rosen p. 86

Idea: To prove A, instead, we prove that the conditional

$$(\neg A) \to F$$

is true. **But**, the only way for a conditional to be true if its conclusion is false is for **its hypothesis to be false too.**

Conclude: $(\neg A)$ is false, i.e. **A is true**!

An (ir)rational excursion

Rosen p. 86, 97

Proof of Lemma: **WTS** $\sqrt{2}$ is not rational.

Assume $\sqrt{2}$ is rational, write it in lowest terms.

Goal: **Contradiction!**

An (ir)rational excursion

Rosen p. 86, 97

Proof of Corollary: **WTS** There are irrational numbers x,y such that x^y is rational.

Possible values of x,y for witness?

Some definitions

Rosen Sections 2.1, 2.2

Empty set: $\emptyset = \{\} = \{x : x \neq x\}$

Which of the following is **not** true?

A. $\emptyset \subseteq \emptyset$

B. $\emptyset \in \emptyset$

C. For any set A, $\emptyset \subseteq A$

D. For some set B, $\emptyset \in B$

E. More than one of the above.

Operations on sets

Rosen Sections 2.1, 2.2

Power set: For a set S, its power set is the set of all subsets of S. $\mathcal{P}(S) = \{A \mid A \subseteq S\}$

Which of the following is **not** necessarily (always) true?

A. $S \in \mathcal{P}(S)$
B. $\emptyset \in \mathcal{P}(S)$
C. $\emptyset \subseteq \mathcal{P}(S)$
D. $\emptyset \in S$
E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

- Given two sets A, B we can define

$A \cap B$	**Intersection of A and B**
$A \cup B$	**Union of A and B**
$A - B$	**Difference of A and B**
$A \times B$	**Cartesian product of A and B**

Operations on sets

Rosen Sections 2.1, 2.2

- Given two sets A, B we can define

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

Operations on sets

Rosen Sections 2.1, 2.2

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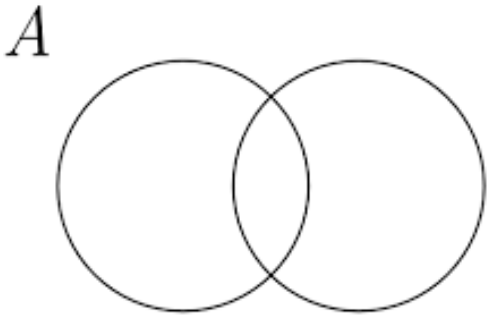

Which of the following **can't** be labelled in this Venn diagram?

A. $A \cap B$

B. $A \cup B$

C. $A - B$

D. $A \times B$

E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

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$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

Which of these is true?

A. $A \cap B = B \cap A$

B. $A \cup B = B \cup A$

C. $A - B = B - A$

D. $A \times B = B \times A$

E. None of the above.