CSE 200
Computability and Complexity
Homework 3
NP, Completeness, and Reductions
Due Tuesday, Feb. 16. Note change

February 4, 2016

Give proofs for each problem. Proofs can be high-level, but be precise. You may use without giving a proof any result proved in class or in the textbook. In particular, to prove NP-completeness, it suffices to give a reduction from any of the NP-complete problems from the text or from class. However, you must show your reduction is valid, by showing the equivalence of the constructed instance and the original.

NP-Completeness: Combinatorial auction Prove that the following problem is NP-complete:

Problem: Maximum Profit Accepted Bids (MPAB)

Instance: A set of n items, $I = \{a_1, ..a_n\}$, and a list of bids, $B_1, ..B_m$ where each bid specifies a subset of the goods desired $S_i \subset I$, and an amount to be paid for these items if the bid is accepted, $p_i \geq 0$. A total value $V > 0$.

Solution space: A subset of the bids to be *accepted*, $A \subseteq \{1, ..m\}$.

Constraints: Each item can be in at most one accepted bid. (i.e., if $S_i \cap S_j \neq \emptyset$, then we cannot have both $i \in A$ and $j \in A$.)

Objective: Decide whether there is a set of accepted bids A meeting the constraints where $\sum_{i \in A} p_i \geq V$.

NP-Completeness: 3-Coloring of degree 5 graphs Show that the 3-coloring problem remains NP-complete when restricted to graphs of degree at most 5.

Sudoku The *sudoku* problem of size n is as follows. The input is an $n^2 \times n^2$ matrix M whose entries are either "blank" or an integer between 1 and n^2. A solution fills in the blank spaces with integers between 1 and n^2.

The following constraints must be met: Each integer from 1 to n^2 appears exactly once in each row, in each column, and in each $n \times n$ sub-matrix of the form $M[jn+1...(j+1)n][in+1..(i+1)n]$ for each $0 \leq i, j \leq n-1$. The problem is to decide whether there is a solution meeting the constraints.

Give at least two distinct reductions from Sodoku to $CNFSAT$. For each, analyze the number of variables in the resulting CNF, and the number of clauses of different sizes. In the next homework, you will be asked to run experiments combining these reductions with a SAT solver to solve Sudoku. Which reduction do you expect to have the best results in the experiment, and why?