CSE 200 Final Exam

Due Wed. March 16 at 11:59 PM (OK, I won't actually check until I get in in Thursday morning. But all exams must be turned in before I arrive in my office.)

Answer all parts of all questions with informal, but complete, proofs. You may not discuss this exam with anyone except myself, whether taking the course or not. Weights are given in the question. You may cite without proof any result from the Arora-Barak text or proved in class. In particular, you can use without proof the NP-completeness of any problem proved NP-complete in class, in the two texts, or on the homeworks, including: SAT, 3-SAT, Independent Set, Clique, Vertex Covering, 3-coloring, Hamiltonian Circuit, and Subset Sum. However, you may not use the list of NP-complete problems in the appendix of Garey and Johnson without proof.

NP-completeness: 10 points Show that the following problem is NP-complete:

Problem: All distinct triples (ADT).

Instance: A set of n elements V, and a set of triples $S_1, ...S_m$, where each $S_i \subset V$ and $|S_i| = 3$ (i.e., the three elements of S_i are all distinct).

Solution format: A coloring of the elements V assigning each one of three colors, R, G and B

Constraints: For any triple S_i, there must be exactly one vertex in S_i of each color.

Objective: Decide whether there is a three coloring of V meeting the constraints, i.e., where all triples have distinct colors.

Note: Set systems over a common universe are sometimes called *hypergraphs*. In this notation, V would be called the set of *vertices* , and the S_i's would be *hyperedges of order 3* . Sorry for mixing notation in the original version.

Closure properties of classes of problems: 5 points each class If L is a decision problem, let ALT_L be the problem of, given a set $x_1, ..x_n$ of instances to L and an integer $0 \leq k \leq n$, deciding whether at least k of the x_i are in L. (ALT stands for "At Least a Threshold"). Say that a class C is closed under ALT if, for any L, if $L \in C$, then $ALT_L \in C$.

Show that the following classes are closed under ALT

1. RE, the class of recursively enumerable (or computably recognizable) languages.
2. NP
3. NL, the class of problems solvable in non-deterministic log space

4. BPP, the class of problems solvable in probabilistic polynomial time

Closure properties of classes: 5 points each class If L is a decision problem, let AMT_L be the problem of, given a set $x_1, ..x_n$ of instances to L and an integer $0 \leq k \leq n$, deciding whether at *most* k of the x_i are in L. (AMT stands for "At Most a Threshold"). Say that a class C is closed under AMT if, for any L, if $L \in C$, then $AMT_L \in C$.

For each of the classes from the previous problem, say whether they are known to be closed under AMT, known not to be closed under AMT, or whether it is an open problem, and explain your answer. (Hint: what other closure property does a class need to have to be closed under AMT?)