

CSE 200  
Computability and Complexity  
Homework 1

Due by email (both to russell@cs.ucsd.edu and jiawei+cse200@ucsd.edu  
) sometime September 29

September 21, 2016

1. Let  $f$  be a non-decreasing, positive integer-valued function over the positive integers. Prove that if  $f(n+1) \in O(f(n))$ , then there is a  $c$  so that  $f(n) \in O(2^{cn})$ . Is the converse always true? Prove it or give a counter-example.
2. Let  $L_N$  be the language of binary strings whose number of 1's is NOT divisible by  $N$ . Prove that, for infinitely many  $N$ 's, the number of states needed by a non-deterministic finite automaton recognizing  $L_N$  is strictly smaller than that of the smallest deterministic finite automaton doing so.
3. Say that you are on a street, and know that the store you are looking for is on the same street, but not how far away or which direction it is. You know it is at least 100 meters away, because you can see 100 meters each way. You will know the store when you get to it. How can you find the store and be sure to walk at most  $cd$  total distance, where  $c$  is a fixed constant that will come from your solution and  $d$  is the distance to the store?