

CSE 140: Components and Design Techniques for Digital Systems

Lecture 6: Universal Gates

CK Cheng

Dept. of Computer Science and Engineering

University of California, San Diego



Combinational Logic: Other Types of Gates

- Universal Set of Gates
 - Motivation, Definition, Examples
- Other Types of Gates
 - 1) XOR
 - 2) NAND / NOR
 - 3) Block Diagram Transfers

Universal Set of Gates: Motivation

- AND, OR, NOT: Logic gates related to reasoning from Aristotle (384-322BCE)
- NAND, NOR: Inverted AND, Inverted OR gates. For VLSI technologies, all gates are inverted (AND,OR operation with a bubble at output).
- Multiplexer + input table: Table based logic for programmability. FPGA technology.
- XOR: Exclusive OR gates. Parity check.

In the future, we may have new sets of gates due to new technologies. Given a set of gates, can the gates in the set cover all possible switching functions?

Universal Set

Universal set is a powerful concept to identify the coverage of a set of gates afforded by a given technology.

Criterion: If the set of gates can implement AND, OR, and NOT gates, the set is universal.



Universal Set Definition

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

{AND, OR, NOT}

{AND, NOT}

{OR, NOT}



Universal Set

Universal Set: A set of gates such that every switching function can be implemented with the gates in this set.

Examples

{AND, OR, NOT}

{AND, NOT} OR can be implemented with AND &

NOT gates: $a+b = (a' b')'$

{OR, NOT} AND can be implemented with OR &

NOT gates: $ab = (a' +b')'$

{XOR} is not universal

{XOR, AND} is universal

iClicker

Is the set {AND, OR} (but no NOT gate) universal?

A. Yes

B. No

iClicker

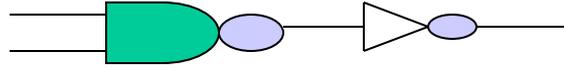
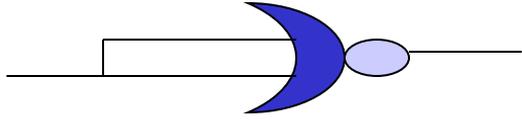
Is the set $\{f(x,y)=xy\}$ universal?

A. Yes

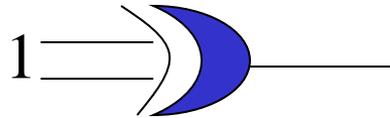
B. No



{NAND, NOR}



{XOR}

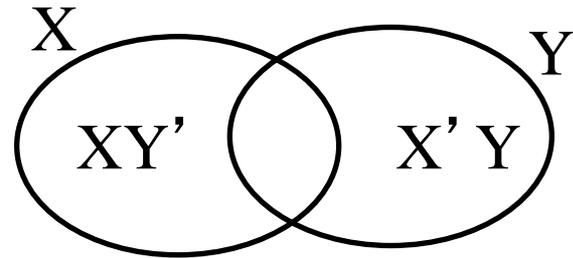


{XOR, AND}

Other Types of Gates: Properties and Usage

1) XOR $X \oplus Y = XY' + X'Y$

It is a parity function (examples)
useful for testing because the
flipping of a single input changes the
output.



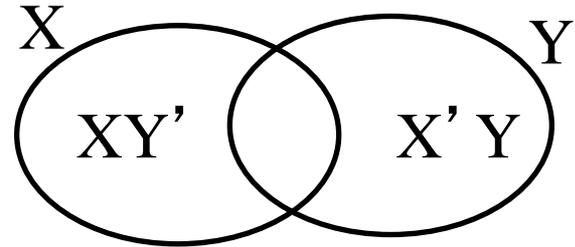
id	x	y	$x \oplus y$
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

	x=0	x=1
y=0	0	1
y=1	1	0



Other Types of Gates: Properties and Usage

1) XOR $X \oplus Y = XY' + X'Y$



(a) Commutative $X \oplus Y = Y \oplus X$

(b) Associative $(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z)$

(c) $1 \oplus X = X'$, $0 \oplus X = 0X' + 0'X = X$

(d) $X \oplus X = 0$, $X \oplus X' = 1$

Other Types of Gates: Properties and Usage

e) If $ab = 0$, then $a \oplus b = a + b$

Proof: If $ab = 0$, then $a = a(b + b') = ab + ab' = ab'$
 $b = b(a + a') = ba + ba' = a'b$
 $a + b = ab' + a'b = a \oplus b$

f) $f(x,y) = x \oplus xy' \oplus x'y \oplus (x + y) \oplus x = ?$
(Priority of operations: AND, \oplus , OR)

Hint: We apply Shannon's Expansion.



Shannon's Expansion (for switching functions)

Formula: $f(X, Y) = X * f(1, Y) + X' * f(0, Y)$

Proof by enumeration:

If $X = 1$, $f(X, Y) = f(1, Y) : 1 * f(1, Y) + 1' * f(0, Y) = f(1, Y)$

If $X = 0$, $f(X, Y) = f(0, Y) : 0 * f(1, Y) + 0' * f(0, Y) = f(0, Y)$



Other types of gates: XOR

Simplify the function (Priority of operations: AND, \oplus , OR)

$$f(X, Y) = X \oplus XY' \oplus X' Y \oplus (X+Y) \oplus X$$

$$\text{Case } X = 1: f(1, Y) = 1 \oplus Y' \oplus 0 \oplus 1 \oplus 1 = Y$$

$$\text{Case } X = 0: f(0, Y) = 0 \oplus 0 \oplus Y \oplus Y \oplus 0 = 0$$

Thus, using Shannon's expansion, we have

$$f(X, Y) = Xf(1, Y) + X'f(0, Y) = XY$$

XOR gates

iClicker: Is the equation

$$a+(b\oplus c) = (a+b)\oplus(a+c) \text{ true?}$$

A. Yes

B. No

Other Types of Gates: NAND, NOR

2) NAND, NOR gates

NAND (NOR) gates are not associative

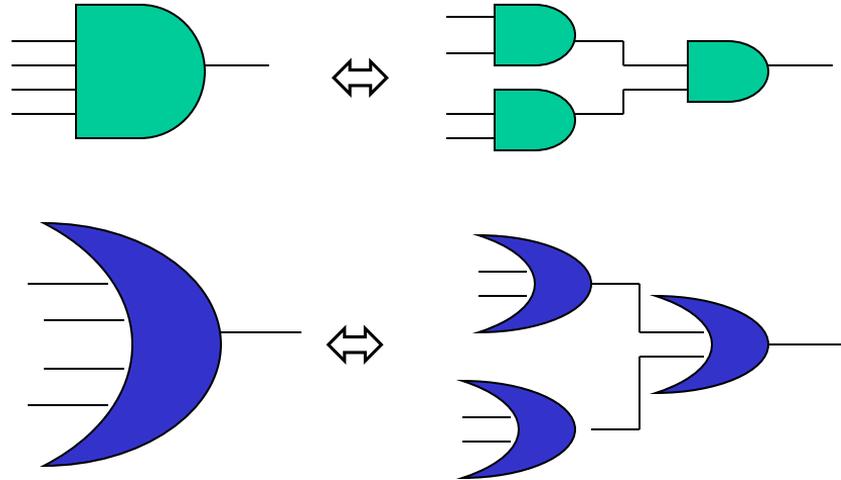
Let $a | b = (ab)'$

$$(a | b) | c \neq a | (b | c)$$

Other Types of Gates: Block Diagram Transform

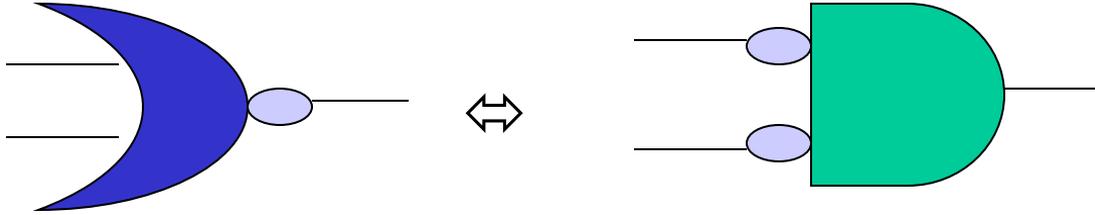
3) Block Diagram Transformation

a) Reduce # of inputs.

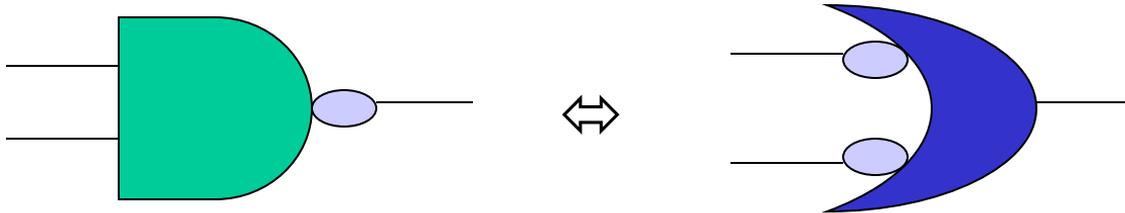


Other Types of Gates: Block Diagram Transform

b. DeMorgan's Law



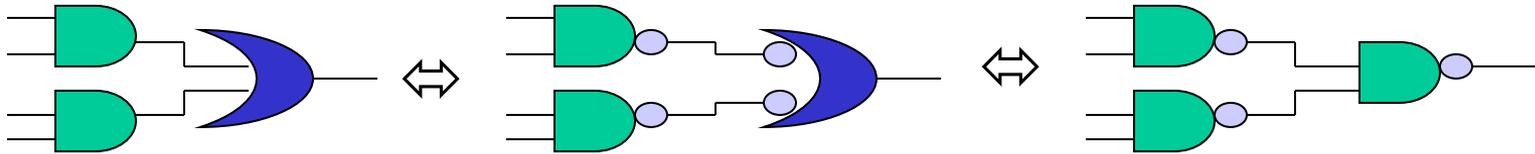
$$(a+b)' = a' b'$$



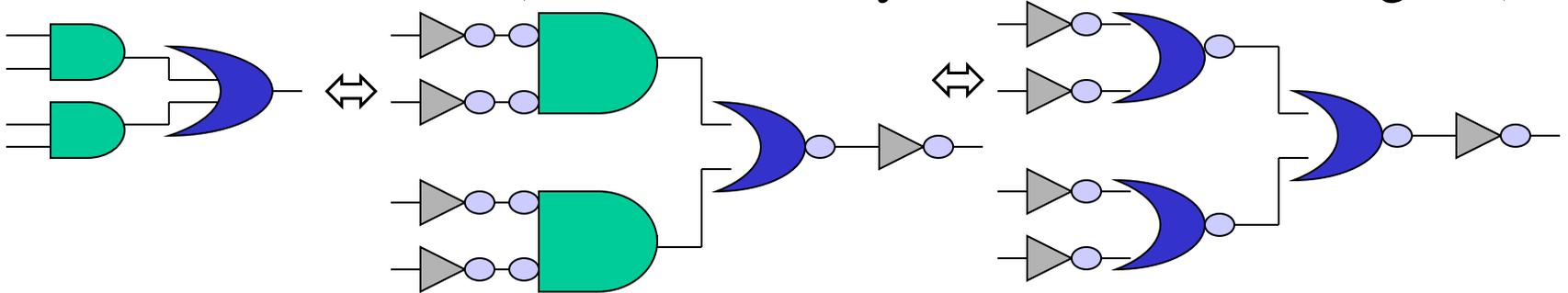
$$(ab)' = a' + b'$$

Other Types of Gates: Block Diagram Transform

c. Sum of Products (Using only NAND gates)



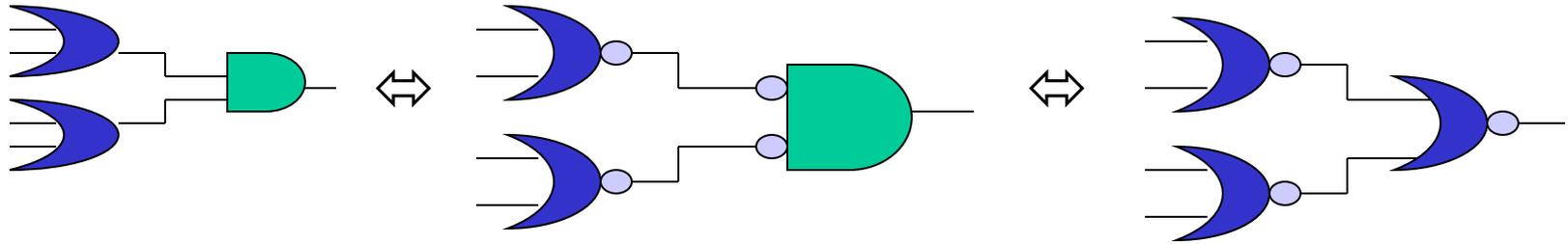
Sum of Products (We create many bubbles with NOR gates)





Other Types of Gates: Block Diagram Transform

d. Product of Sums (NOR gates only)



We will create many bubbles with NAND gates.

Other Types of Gates: Block Diagram Transform

NAND, NOR gates

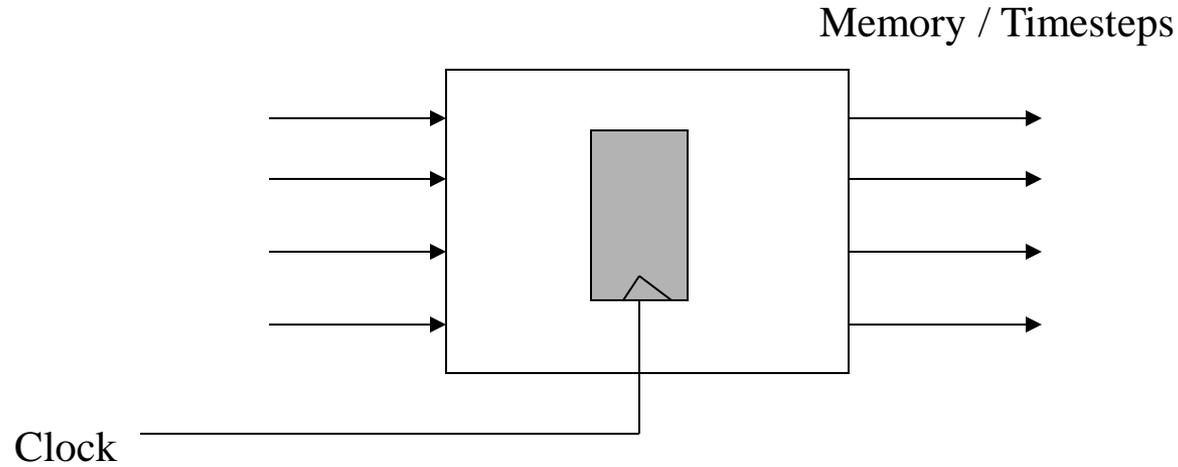
Remark:

Two level NAND gates: Sum of Products

Two level NOR gates: Product of Sums



Part II. Sequential Networks



Flip flops
Specification
Implementation