Lecture 5: K-Map minimization in larger input dimensions and K-map minimization using max terms

CSE 140: Components and Design Techniques for Digital Systems

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Part I. Combinational Logic

1. Specification
2. Implementation

   K-map:

   Sum of products

   Product of sums

Implicant: A product term that has non-empty intersection with on-set $\mathcal{F}$ and does not intersect with off-set $\mathcal{R}$.

Prime Implicant: An implicant that is not a proper subset of any other implicant.

Essential Prime Implicant: A prime implicant that has an element in on-set $\mathcal{F}$ but this element is not covered by any other prime implicants.

Implicate: A sum term that has non-empty intersection with off-set $\mathcal{R}$ and does not intersect with on-set $\mathcal{F}$.

Prime Implicate: An implicate that is not a proper subset of any other implicate.

Essential Prime Implicate: A prime implicate that has an element in off-set $\mathcal{R}$ but this element is not covered by any other prime implicates.

K-Map to Minimized Product of Sums

- Sometimes easier to reduce the K-map by considering the offset
  - Usually when number of zero outputs is less than number of outputs that evaluate to one OR offset is smaller than onset

cd \ ab	00	01	11	10
00	1	1	1	1
01	0	1	1	1
11	0	1	1	1
10	1	1	1	1

Minimum Sum of Products

Given $\mathcal{F}(a,b,c) = \Sigma m\ (3, 5)$, $\mathcal{D} = \Sigma m\ (0, 4)$

c \ ab	00	01	11	10
0	0	2	6	4
1	1	3	7	5

Prime Implicants:

Essential Prime Implicants:

Min SOP exp: f(a,b,c)=

Minimum Sum of Products

Given $\mathcal{F}$(a,b,c) = Σm (3, 5), $\mathcal{D}$ = Σm (0, 4)

c \ ab	00	01	11	10
0	0: X	2: 0	6: 0	4: X
1	1: 0	3: 1	7: 0	5: 1

Prime Implicants: Σm (3), Σm (4, 5)

Essential Prime Implicants: Σm (3), Σm (4, 5)

Min SOP exp: f(a,b,c) = a' bc + ab'

Minimum Product of Sums

Given $\mathcal{F}(a,b,c) = \Sigma m\ (3, 5)$, $\mathcal{D} = \Sigma m\ (0, 4)$

c \ ab	00	01	11	10
0	0: X	2: 0	6: 0	4: X
1	1: 0	3: 1	7: 0	5: 1

Minimum Product of Sums

Given $\mathcal{F}(a,b,c) = \Sigma m\,(3, 5)$, $\mathcal{D} = \Sigma m\,(0, 4)$

c \ ab	00	01	11	10
0	(0) X	(2) 0	(6) 0	(4) X
1	(1) 0	(3) 1	(7) 0	(5) 1

$\overline{\mathcal{F}}(a,b,c) = \Sigma m\,(1, 2, 6,7) + \Sigma d\,(0, 4)$

c \ ab	00	01	11	10
0	(0)	(2)	(6)	(4)
1	(1)	(3)	(7)	(5)

Minimum Product of Sum: Boolean Algebra Rationale

$\mathcal{F}(a,b,c) = \Sigma m\,(1, 2, 6, 7) + \Sigma d\,(0, 4)$

c \ ab	00	01	11	10
0	[0] X	[2] 1	[6] 1	[4] X
1	[1] 1	[3] 0	[7] 1	[5] 0

Minimum Product of Sums

Given $\mathcal{F}(a,b,c) = \Sigma m\ (3, 5)$, $\mathcal{D} = \Sigma m\ (0, 4)$

c \ ab	00	01	11	10
0	0: X	2: 0	6: 0	4: X
1	1: 0	3: 1	7: 0	5: 1

Minimum Product of Sums

Given $\mathcal{F}(a,b,c) = \Sigma m\ (3, 5)$, $\mathcal{D} = \Sigma m\ (0, 4)$

c \ ab	00	01	11	10
0	0: X	2: 0	6: 0	4: X
1	1: 0	3: 1	7: 0	5: 1

PI Q: The adjacent cells grouped in red can be minimized to the following max term:

A. a+b

B. (a+b)’

C. a’+b’

Minimum Product of Sums

Given $\mathcal{F}(a,b,c) = \Sigma m\ (3, 5)$, $\mathcal{D} = \Sigma m\ (0, 4)$

c \ ab	00	01	11	10
0	(0) X	(2) 0	(6) 0	(4) X
1	(1) 0	(3) 1	(7) 0	(5) 1

Prime Implicates:

Essential Primes Implicates:

Min exp: f(a,b,c) =

Minimum Product of Sums

Given $\mathcal{F}$ (a,b,c) = Σm (3, 5), $\mathcal{D}$ = Σm (0, 4)

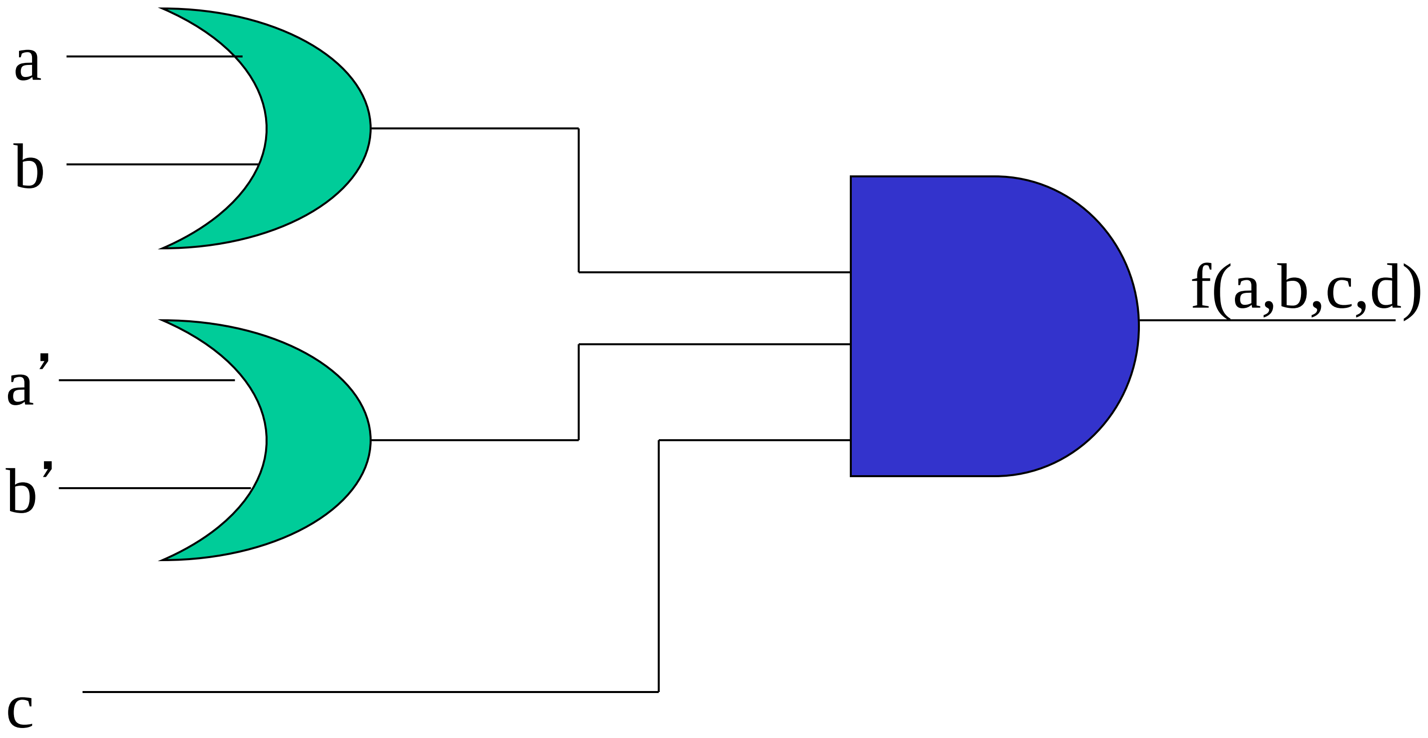
c \ ab	00	01	11	10
0	0: X	2: 0	6: 0	4: X
1	1: 0	3: 1	7: 0	5: 1

Prime Implicates: ΠM (0, 1), ΠM (0, 2, 4, 6), ΠM (6, 7)

Essential Primes Implicates: ΠM (0, 1), ΠM (0, 2, 4, 6), ΠM(6, 7)

Min exp: f(a,b,c) = (a+b)(c)(a' +b')

Corresponding Circuit



Another min product of sums example

Given $\mathcal{R}(a,b,c,d) = \Sigma m\ (3, 11, 12, 13, 14)$

$\mathcal{D}\ (a,b,c,d) = \Sigma m\ (4, 8, 10)$

K-map

cd \ ab	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Another min product of sums example

Given $\mathcal{R}(a,b,c,d) = \Sigma m\ (3, 11, 12, 13, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m\ (4, 8, 10)$

Is $\pi M(12,14)$ a prime implicate?

A: Yes

B: No

cd \ ab	00	01	11	10
00	(0) 1	(4) X	(12) 0	(8) X
01	(1) 1	(5) 1	(13) 0	(9) 1
11	(3) 0	(7) 1	(15) 1	(11) 0
10	(2) 1	(6) 1	(14) 0	(10) X

d

a

Prime Implicates: ΠM (3,11), ΠM (12,13), ΠM(10,11), ΠM (4,12), ΠM (8,10,12,14)

PI Q: Which of the following is a non-essential prime implicate?

A. ΠM(3,11)

B. ΠM(12,13)

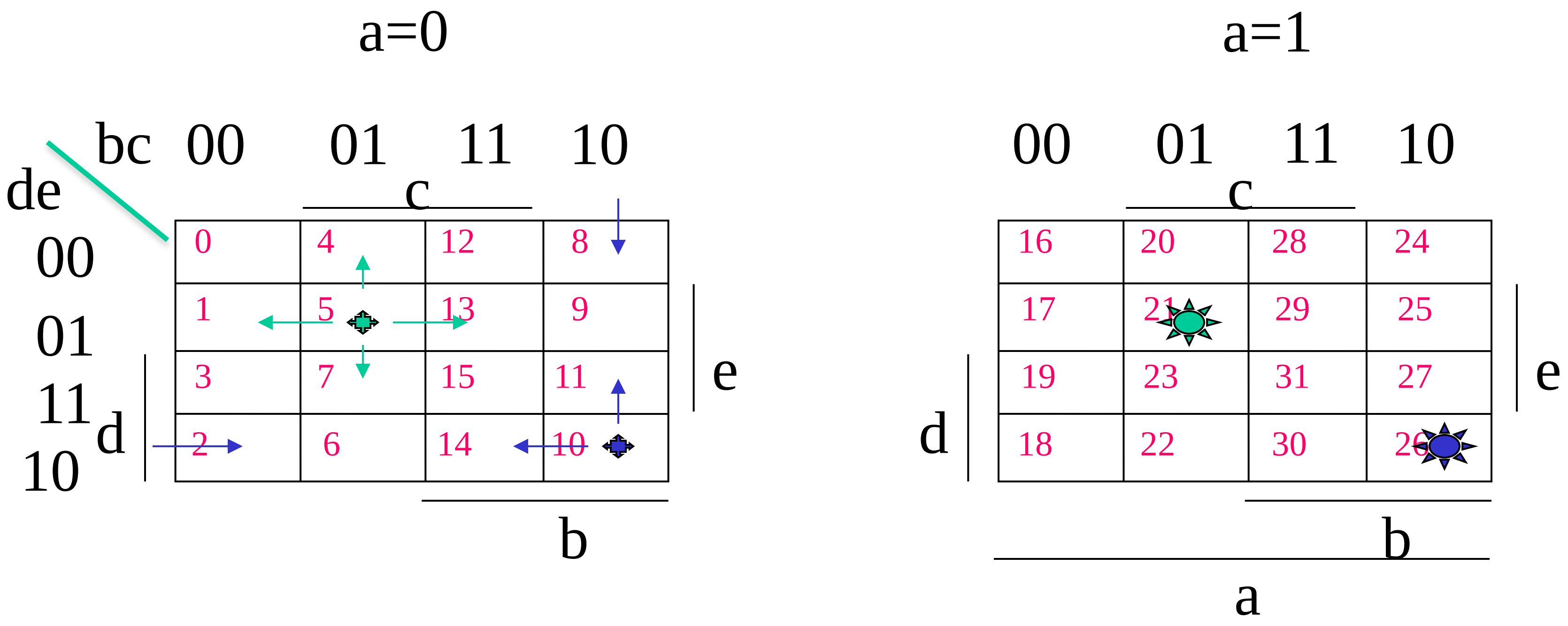
C. ΠM(10,11)

D. ΠM(8,10,12,14)

cd \ ab	00	01	11	10
00	0: 1	4: X	12: 0	8: X
01	1: 1	5: 1	13: 0	9: 1
11	3: 0	7: 1	15: 1	11: 0
10	2: 1	6: 1	14: 0	10: X

d (rows 01, 11); a (columns 11, 10)

Five variable K-map



Neighbors of m_5 are: minterms 1, 4, 7, 13, and 21

Neighbors of m_{10} are: minterms 2, 8, 11, 14, and 26

Reading a Five variable K-map

a=0

de \ bc	00	01	11	10
00	0: 1	4	12: 1	8: 1
01	1	5: 1	13: 1	9
11	3	7	15: 1	11: 1
10	2: 1	6	14: 1	10: 1

c (columns 01, 11); b (columns 11, 10); d (rows 11, 10); e (rows 01, 11)

a=1

de \ bc	00	01	11	10
00	16: 1	20	28	24: 1
01	17	21	29	25
11	19	23: 1	31: 1	27: 1
10	18: 1	22	30: 1	26: 1

c (columns 01, 11); b (columns 11, 10); d (rows 11, 10); e (rows 01, 11); a (whole map)

Six variable K-map

d

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

e f c

d

16	20	28	24
17	21	29	25
19	23	31	27
18	22	30	26

e f c

d

32	36	44	40
33	37	45	41
35	39	47	43
34	38	46	42

e f c

d

48	52	60	56
49	53	61	57
51	55	63	59
50	54	62	58

e f a c

b