Lecture 4: Four Input K-Maps

CSE 140: Components and Design Techniques for Digital Systems

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Outlines

- Boolean Algebra vs. Karnaugh Maps
  - Algebra: variables, product terms, minterms, consensus theorem
  - Map: planes, rectangles, cells, adjacency
- Definitions: implicants, prime implicants, essential prime implicants
- Implementation Procedures

4-input K-map

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Y

CD \ AB	00	01	11	10
00				
01				
11				
10				

4-input K-map

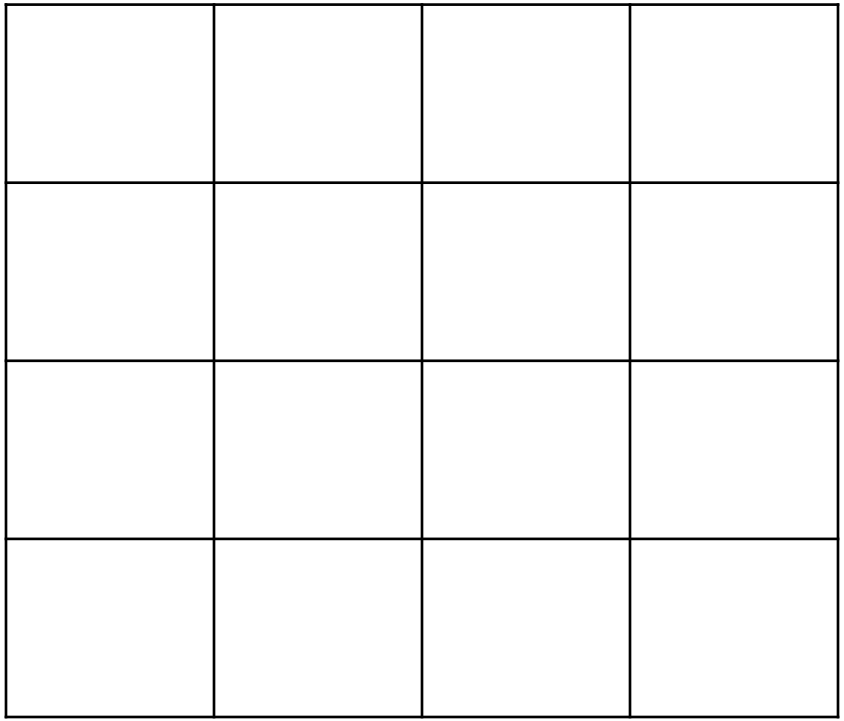
A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Y: CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

4-input K-map

- Arrangement of variables
- Adjacency and partition

Y CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1



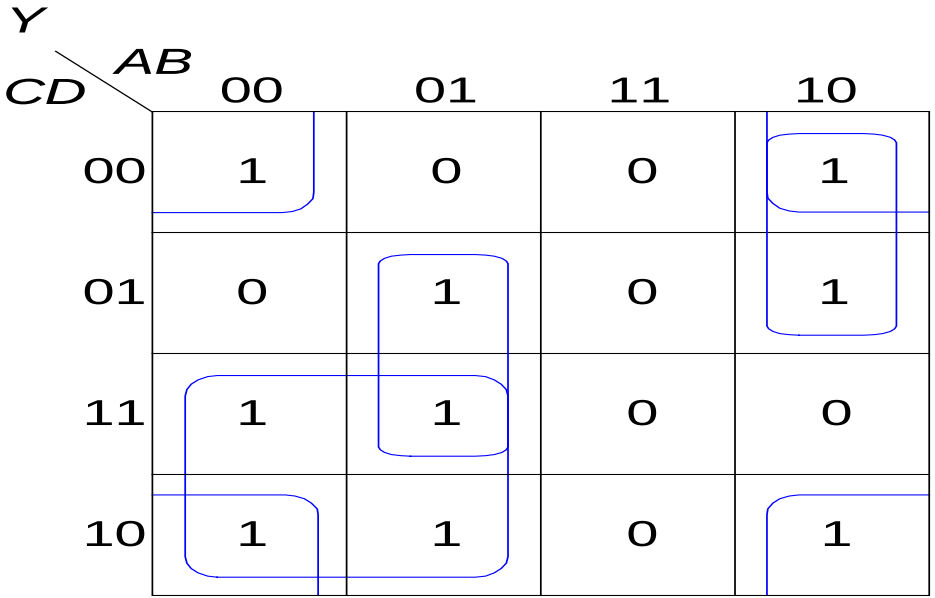
Boolean Expression	K-Map
Variable x_i and complement x_i'	⇔ Half planes Rx_i, and Rx_i'
Product term $P=\prod_i x_i$ *	⇔ Intersect of Rx_i* for all i in P
Each minterm	⇔ One element cell
Two minterms are adjacent.	⇔ The two cells are neighbors
Each minterm has n adjacent minterms	⇔ Each cell has n neighbors

Procedure for finding the minimal function via K-maps (layman terms)

1. Convert truth table to K-map
2. Group adjacent ones: In doing so include the largest number of adjacent ones (Prime Implicants)
3. Create new groups to cover all ones in the map: create a new group only to include at least one cell (of value 1) that is not covered by any other group
4. Select the groups that result in the minimal sum of products (we will formalize this because its not straightforward)

Y / CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

Reading the reduced K-map



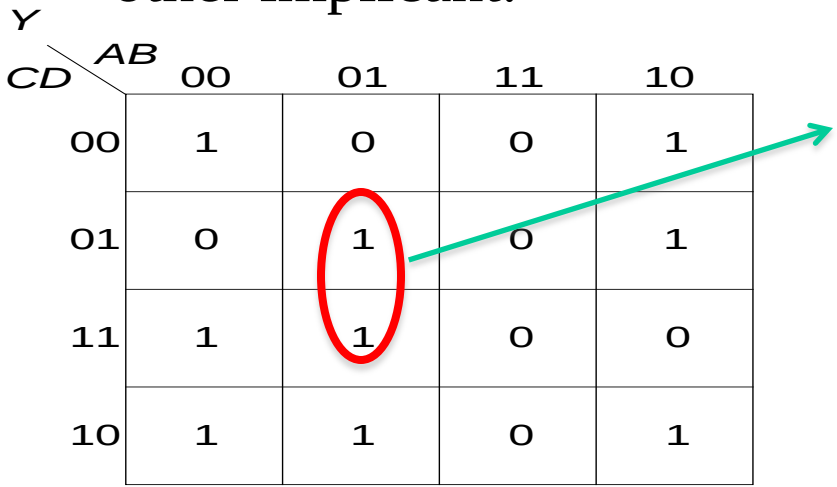
$$Y = \overline{A}C + \overline{A}B\overline{D} + A\overline{B}\overline{C} + \overline{B}\overline{D}$$

Definitions: implicant, prime implicant, essential prime implicant

- Implicant: A product term that has non-empty intersection with on-set *F* and does not intersect with off-set *R* .
- Prime Implicant: An implicant that is not covered by any other implicant.
- Essential Prime Implicant: A prime implicant that has an element in on-set *F* but this element is not covered by any other prime implicants.

Definition: Prime Implicant

1. Implicant: A product term that has non-empty intersection with on-set *F* and does not intersect with off-set *R*.
2. Prime Implicant: An implicant that is not covered by any other implicant.

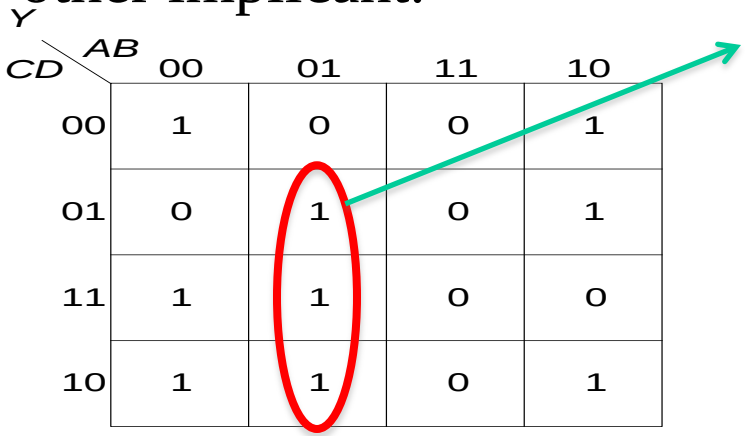


Q: Is this a prime implicant?

A. Yes
B. No

Definition: Prime Implicant

1. Implicant: A product term that has non-empty intersection with on-set *F* and does not intersect with off-set *R*.
2. Prime Implicant: An implicant that is not covered by any other implicant.

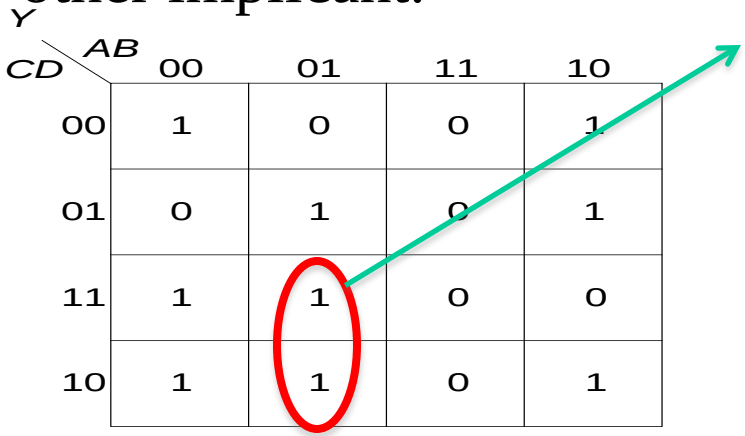


Q: How about this one? Is it a prime implicant?

A. Yes
B. No

Definition: Prime Implicant

1. Implicant: A product term that has non-empty intersection with on-set *F* and does not intersect with off-set *R*.
2. Prime Implicant: An implicant that is not covered by any other implicant.

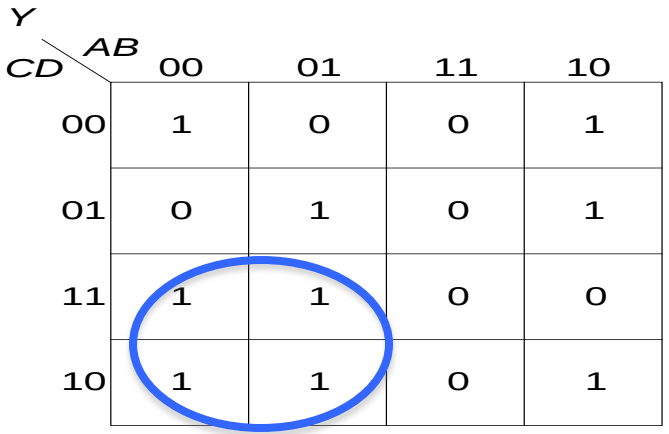


Q: How about this one? Is it a prime implicant?

A. Yes
B. No

Definition: Essential Prime

- Essential Prime Implicant: A prime implicant that has an element in on-set F but this element is not covered by any other prime implicants.



Q: Is the blue group an essential prime?

A. Yes

B. No

Definition: Non-Essential Prime

Non Essential Prime Implicant : Prime implicant that has no element that cannot be covered by other prime implicant

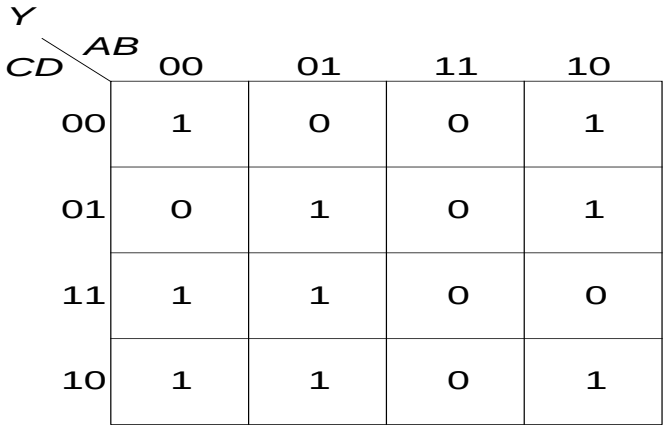
Q: Which of the following reduced expressions is obtained from a non-essential prime for the given K-map ?

cd \ ab	00	01	11	10
00	1		1	1
01		1	1	
11			1	1
10	1		1	1

A. bc'd
B. d'b'
C. ac
D. abc
E. ad'

Procedure for finding the minimal function via K-maps (formal terms)

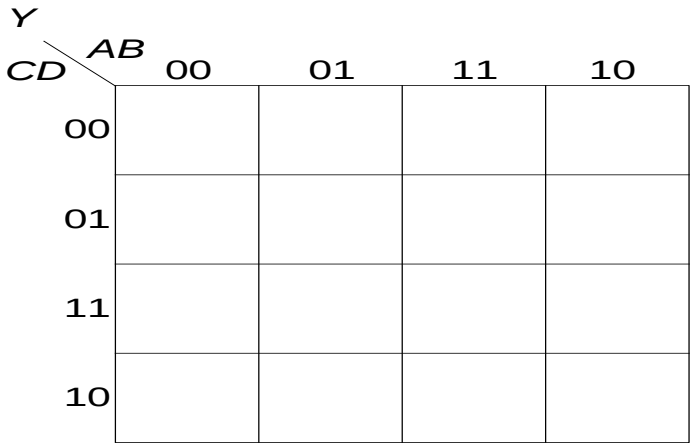
1. Convert truth table to K-map
2. Include all essential primes
3. Include non essential primes as needed to completely cover the onset (all cells of value one)



Y	AB 00	01	11	10
CD 00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

K-maps with Don’t Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X



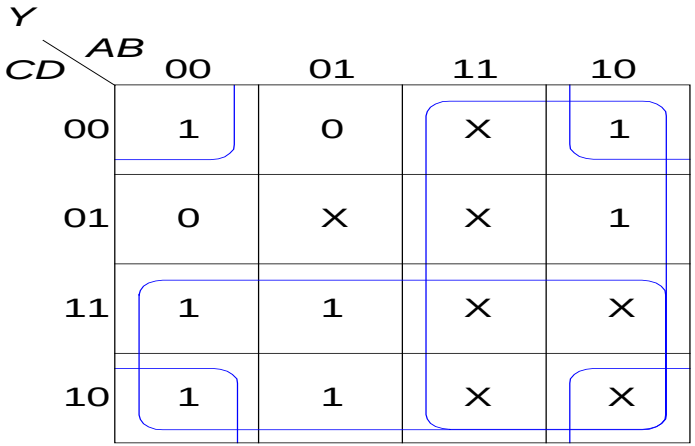
K-maps with Don’t Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y: CD \ AB	00	01	11	10
00	1	0	X	1
01	0	X	X	1
11	1	1	X	X
10	1	1	X	X

K-maps with Don’t Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X



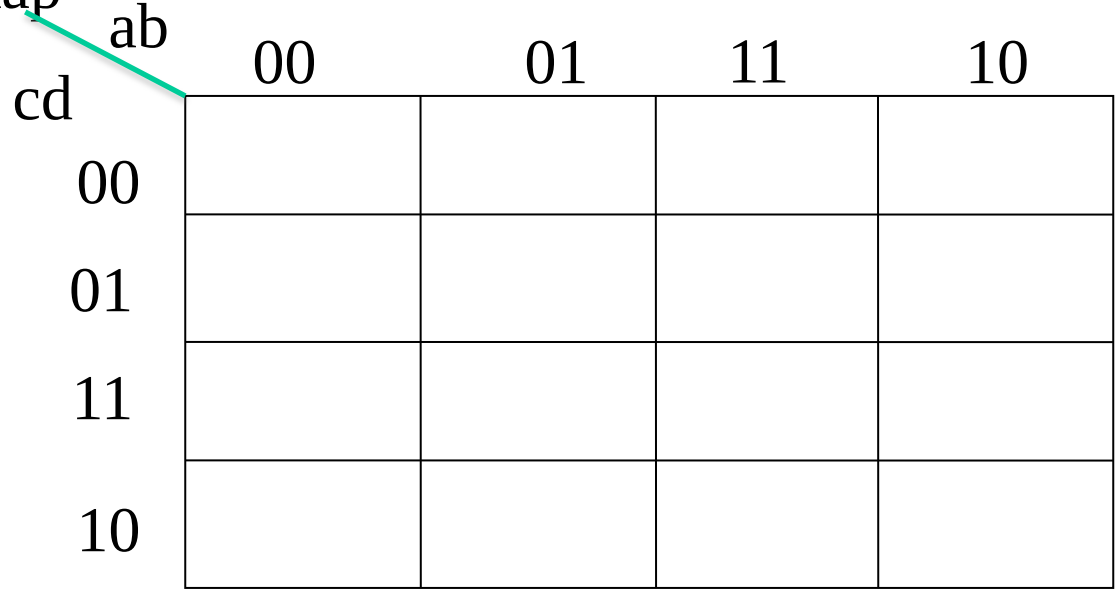
$Y = A + \bar{B}\bar{D} + C$

Reducing Canonical expressions

Given $\mathcal{F}(a,b,c,d) = \Sigma m\ (0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m\ (9, 10)$

1. Draw K-map



Reducing Canonical Expressions

Given $\mathcal{F}(a,b,c,d) = \Sigma m\ (0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m\ (9, 10)$

1. Draw K-map

cd \ ab	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Reducing Canonical Expressions

Given $\mathcal{F}(a,b,c,d) = \Sigma m\,(0, 1, 2, 8, 14)$

$\mathcal{D}(a,b,c,d) = \Sigma m\,(9, 10)$

1. Draw K-map

cd \ ab	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 0	13: 0	9: X
11	3: 0	7: 0	15: 0	11: 0
10	2: 1	6: 0	14: 1	10: X

Reducing Canonical Expressions

1. Draw K-map
2. Identify Prime implicants
3. Identify Essential Primes

cd \ ab	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 0	13: 0	9: X
11	3: 0	7: 0	15: 0	11: 0
10	2: 1	6: 0	14: 1	10: X

PI Q: How many primes (P) and essential primes (EP) are there?

A. Four (P) and three (EP)
B. Three (P) and two (EP)
C. Three (P) and three (EP)
D. Four (P) and Four (EP)

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
2. Essential Primes: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)

cd \ ab	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 1	5 0	13 0	9 X
11	3 0	7 0	15 0	11 0
10	2 1	6 0	14 1	10 X

PI Q: Do the E-primes cover the entire on set?

A. Yes
B. No

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
2. Essential Primes: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
3. Min exp: Σ (Essential Primes)=Σm (0, 1, 8, 9) + Σm (0, 2, 8, 10) + Σm (10, 14)

f(a,b,c,d) = ?

cd \ ab	00	01	11	10
00	0: 1	4: 0	12: 0	8: 1
01	1: 1	5: 0	13: 0	9: X
11	3: 0	7: 0	15: 0	11: 0
10	2: 1	6: 0	14: 1	10: X

PI Q: Do the E-primes cover the entire on set?

A. Yes

B. No

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
2. Essential Primes: Σm (0, 1, 8, 9), Σm (0, 2, 8, 10), Σm (10, 14)
3. Min exp: Σ (Essential Primes)=Σm (0, 1, 8, 9) + Σm (0, 2, 8, 10) + Σm (10, 14)

f(a,b,c,d) = b' c' + b' d' + acd '

cd \ ab	00	01	11	10
00	0 1	4 0	12 0	8 1
01	1 1	5 0	13 0	9 X
11	3 0	7 0	15 0	11 0
10	2 1	6 0	14 1	10 X

PI Q: Do the E-primes cover the entire on set?

A. Yes

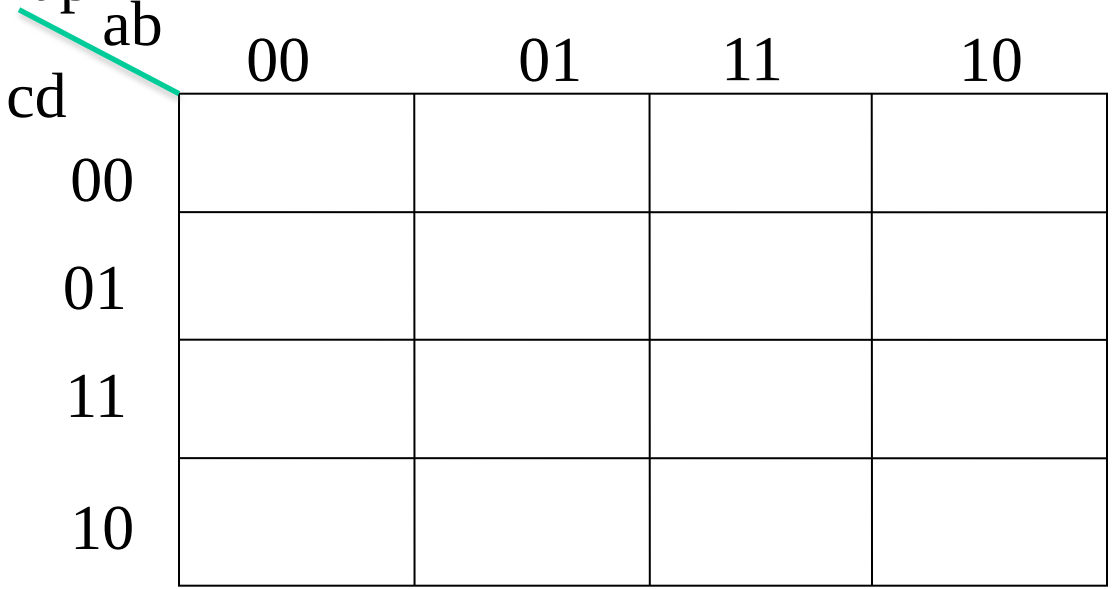
B. No

Another example

Given $F(a,b,c,d) = \Sigma m\ (0, 3, 4, 14, 15)$

$D(a,b,c,d) = \Sigma m\ (1, 11, 13)$

1.Draw the K-Map



Another example

Given $F(a,b,c,d) = \Sigma m\,(0, 3, 4, 14, 15)$

$D(a,b,c,d) = \Sigma m\,(1, 11, 13)$

cd \ ab	00	01	11	10
00	0 1	4 1	12 0	8 0
01	1 X	5 0	13 X	9 0
11	3 1	7 0	15 1	11 X
10	2 0	6 0	14 1	10 0

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 4), Σm (0, 1), Σm (1, 3), Σm (3, 11), Σm (14, 15), Σm (11, 15), Σm (13, 15)

2. Essential Primes: Σm (0, 4), Σm (14, 15)

cd \ ab	00	01	11	10
00	0: 1	4: 1	12: 0	8: 0
01	1: X	5: 0	13: X	9: 0
11	3: 1	7: 0	15: 1	11: X
10	2: 0	6: 0	14: 1	10: 0

Reducing Canonical Expressions

1. Prime implicants: Σm (0, 4), Σm (0, 1), Σm (1, 3), Σm (3, 11), Σm (14, 15), Σm (11, 15), Σm (13, 15)
2. Essential Primes: Σm (0, 4), Σm (14, 15)
3. Min exp: Σm (0, 4), Σm (14, 15), (Σm (3, 11) or Σm (1,3))
4. f(a,b,c,d) = a'c'd'+ abc+ b'cd (or a'b'd)

cd \ ab	00	01	11	10
00	0 1	4 1	12 0	8 0
01	1 X	5 0	13 X	9 0
11	3 1	7 0	15 1	11 X
10	2 0	6 0	14 1	10 0