# HW#3, Solution

**PROBLEM 2 (Universal gates)**

For showing if a set of gates is universal or not, we need to check if we can build the AND, OR and NOT functionality using the set.

1. {XOR, NOT} , is **not universal.**
   - NOT : X xor 1 = X' = NOT X
   - AND : AND gate cannot be built using only {XOR, NOT}
   - OR : OR gate cannot be built using only {XOR, NOT}
2. {NAND, NOT}, is **universal.**
   - NOT : is given
   - AND : (X NAND Y)' = X AND Y
   - OR : (X' NAND Y') = (X'Y')' = X + Y = X OR Y
3. {XNOR}, is **not universal.**
   - NOT : X xnor 0 = X' = NOT X
   - AND : AND gate cannot be built using only {XNOR}
   - OR : OR gate cannot be build using only {XNOR}
4. {f(a, b)}, where f(a, b) = a ′ + b ′ , is **universal.**
   - NOT : f ( X , 1 ) = X' + 0 = X' = NOT X
   - AND : f (X , Y)' = (X' + Y')' = XY = X AND Y
   - OR : f (X', Y') = (X + Y) = X OR Y
5. {f(a, b, c)}, where f(a, b, c) = (a + b)(b + c)(a ′ + b ′ + c ′ ) , is **universal.**
   - NOT : f(a , 1 , 1 ) = (a + 1)(1 + 1)(a ′ + 0 + 0 ) = a' = NOT a
   - AND : f(a,1,c)'=(a +1)(1 + c)(a ′ + 0 + c ′ ) = ac = a AND c
   - OR : f(a',1,c')= (a' + 1)(1 + c')(a + 0 + c ) = a+c = a OR c
6. {f(a, b, c)}, where f(a, b, c) = ab + ac + bc , is **not universal.**
   - NOT : NOT gate cannot be build with given function
   - AND : f(a,b,0) = ab + a0 + b0= ab = a AND b
   - OR :f(a,b,1) = ab + a1 + b1= a+b = a OR b

**PROBLEM 3 (Other types of gates)**

I. simplify expressions

1. f(a,b) = a ⊕ (a + b ′ ) ⊕ ab ⊕ a ′ b

   We can simplify the equation using shannon theorem:

   shannon formula : f(a,b) = a.f(1,b) + a'f(0,b)

   f(1,b) = 1 ⊕ (1 + b ′ ) ⊕ 1b ⊕ 0 b

   = 1 ⊕ 1 ⊕ b ⊕ 0

$= 0 \oplus b$
$= b$
$f(0,b) = 0 \oplus (0 + b') \oplus 0b \oplus 1\, b$
$= 0 \oplus b' \oplus 0 \oplus b = b \oplus b' = 1$

substituting in shannon formula :
$f(a,b) = a.f(1,b) + a'f(0,b) = ab + a'$
$= ab + a' + b$ //consensus
$= b\,(a+1) + a'$
$= b + a'$

2. $f(a, b, c) = abc \oplus (a + b' + c) \oplus (b + c)b \oplus a'b \oplus (b + c')$
We can simplify the equation using shannon theorem:
Shannon formula: $f(a,b,c) = b.f(a,1,c) + b'f(a,0,c)$
$f(a,1,c) = abc \oplus (a + b' + c) \oplus (b + c)b \oplus a'b \oplus (b + c')$
$= a1c \oplus (a + 0 + c) \oplus (1 + c)1 \oplus a'1 \oplus (1 + c')$
$= ac \oplus (a + c) \oplus 1 \oplus a' \oplus 1$
$= ac \oplus (a + c) \oplus a'$ //$f(a,c) = a\, f(1,c) + a'\, f(0,c)$
$= a\,(1c \oplus (1 + c) \oplus 0) + a'(0c \oplus (0 + c) \oplus 1)$
$= a\,(c \oplus 1 \oplus 0) + a'(0 \oplus c \oplus 1)$
$= a\,(c \oplus 1) + a'(c \oplus 1)$
$= a\,(c') + a'(c')$
$= ac' + a'c'$
$= c'(a + a')$
$= c'$
$f(a,0,c) = a0c \oplus (a + 1 + c) \oplus (0 + c)0 \oplus a'0 \oplus (0 + c')$
$= 0 \oplus 1 \oplus 0 \oplus 0 \oplus c'$
$= 1 \oplus c'$
$= c$
$f(a,b,c) = b.f(a,1,c) + b'f(a,0,c)$
$= bc' + b'c$
$= b \oplus c$

II. Prove or disprove the following equalities.

1. $a \oplus bc = (a \oplus b)(a \oplus c)$
we can show the equality does not hold by a counterexample :
if $b=1$, $c=0$
LHS $= a \oplus bc = a \oplus 0 = a$
RHS $= (a \oplus b)(a \oplus c) = (a \oplus 1)(a \oplus 0) = (a')\,(a) = 0$
RHS $\neq$ LHS.

LHS $= a(bc) + a'(bc)$

RHS = (ab' + a'b ) ( ac' + a'c)
= ab'c' + a'bc
RHS ≠ LHS.

So, we cannot distribute XOR over AND.

2. a(b ⊕ c) = ab ⊕ ac

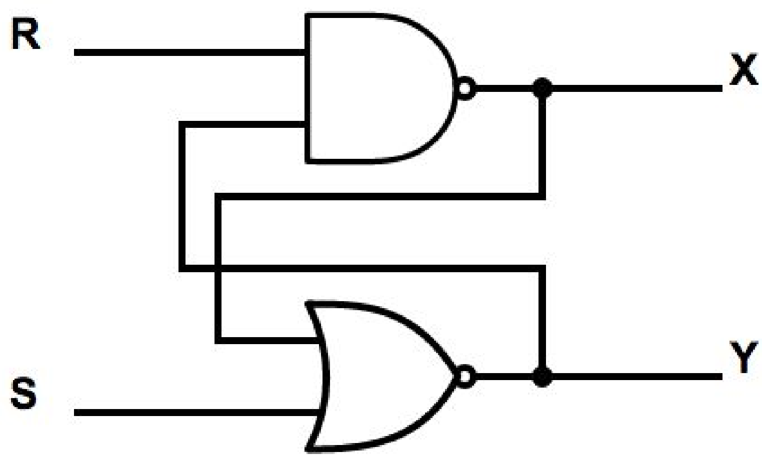
RHS= ab ⊕ ac =(ab)'(ac) + (ab)(ac)'
= (a'+b')(ac) + (ab)(a'+c')
= a'ac+ b'ac +aba' + abc'
= 0 + b'ac + 0+ abc'
= abc'+ ab'c
= a(bc' + b'c)
=a(b ⊕ c)
=LHS

So, we can distribute AND over XOR.

**PROBLEM 4 (Latch)**

**A. Replacing the top NOR gate with a NAND gate**

**X(t+1) = (R Y(t))'**
**Y(t+1) = (S + X(t))'**




| id | X(t) | Y(t) | S | R | X(t1) | Y(t1) | X(t2) | Y(t2) | X(t3) | Y(t3) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |

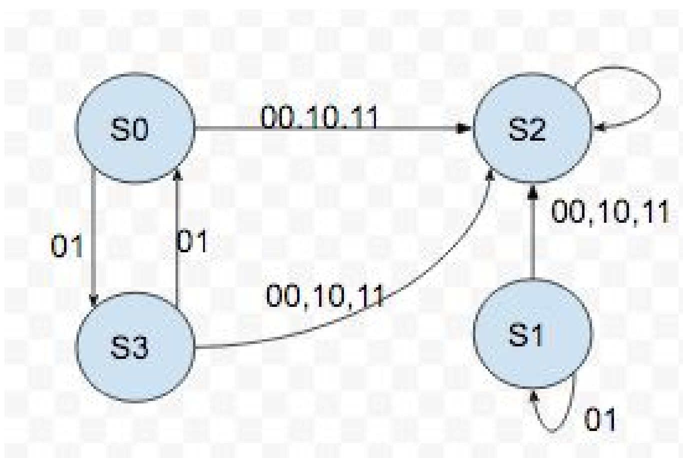
| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 4 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 5 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 7 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 9 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 10 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 11 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 13 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 14 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |

**using S0 = 00, S1= 01, S2 = 10, S3=11**

| id | Current state | S | R | state (t1) | state X(t2) | state X(t3) |
|---|---|---|---|---|---|---|
| 0 | S0 | 0 | 0 | S3 | S2 | S2 |
| 1 | S0 | 0 | 1 | S3 | S0 | S3 |
| 2 | S0 | 1 | 0 | S2 | S2 | S2 |
| 3 | S0 | 1 | 1 | S2 | S2 | S2 |
| 4 | S1 | 0 | 0 | S3 | S2 | S2 |
| 5 | S1 | 0 | 1 | S1 | S1 | S1 |
| 6 | S1 | 1 | 0 | S2 | S2 | S2 |
| 7 | S1 | 1 | 1 | S0 | S2 | S2 |

| | | | | | | |
|---|---|---|---|---|---|---|
| **8** | **S2** | **0** | **0** | **S2** | **S2** | **S2** |
| **9** | **S2** | **0** | **1** | **S2** | **S2** | **S2** |
| **10** | **S2** | **1** | **0** | **S2** | **S2** | **S2** |
| **11** | **S2** | **1** | **1** | **S2** | **S2** | **S2** |
| **12** | **S3** | **0** | **0** | **S2** | **S2** | **S2** |
| **13** | **S3** | **0** | **1** | **S0** | **S3** | **S0** |
| **14** | **S3** | **1** | **0** | **S2** | **S2** | **S2** |
| **15** | **S3** | **1** | **1** | **S0** | **S2** | **S2** |

**state diagram:**




**we can avoid the input "01" and ignore states S0 and S3 to prevent the unstable loop.**

**next state:**

| **current state/ SR** | **00** | **01** | **11** | **10** |
|---|---|---|---|---|
| **01 (s1)** | **s2** | **x** | **s2** | **s2** |
| **10 (s2)** | **s2** | **x** | **s2** | **s2** |

| current state/ SR | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 01 (s1) | 10 | x | 10 | 10 |
| 10 (s2) | 10 | x | 10 | 10 |

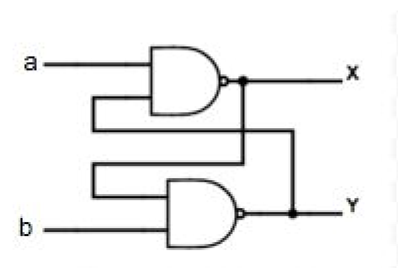
X (t+1) = 1

| current state/ SR | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 01 (s1) | 1 | x | 1 | 1 |
| 10 (s2) | 1 | x | 1 | 1 |

y (t+1) = 0

| current state/ SR | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 01 (s1) | 0 | x | 0 | 0 |
| 10 (s2) | 0 | x | 0 | 0 |

**B. Replacing both NOR gates with NAND gates**




X(t+1) = (a Y(t))'
Y(t+1) = (b X(t))'

| id | X(t) | Y(t) | b | a | X(t1) | Y(t1) | X(t2) | Y(t2) | X(t3) | Y(t3) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |

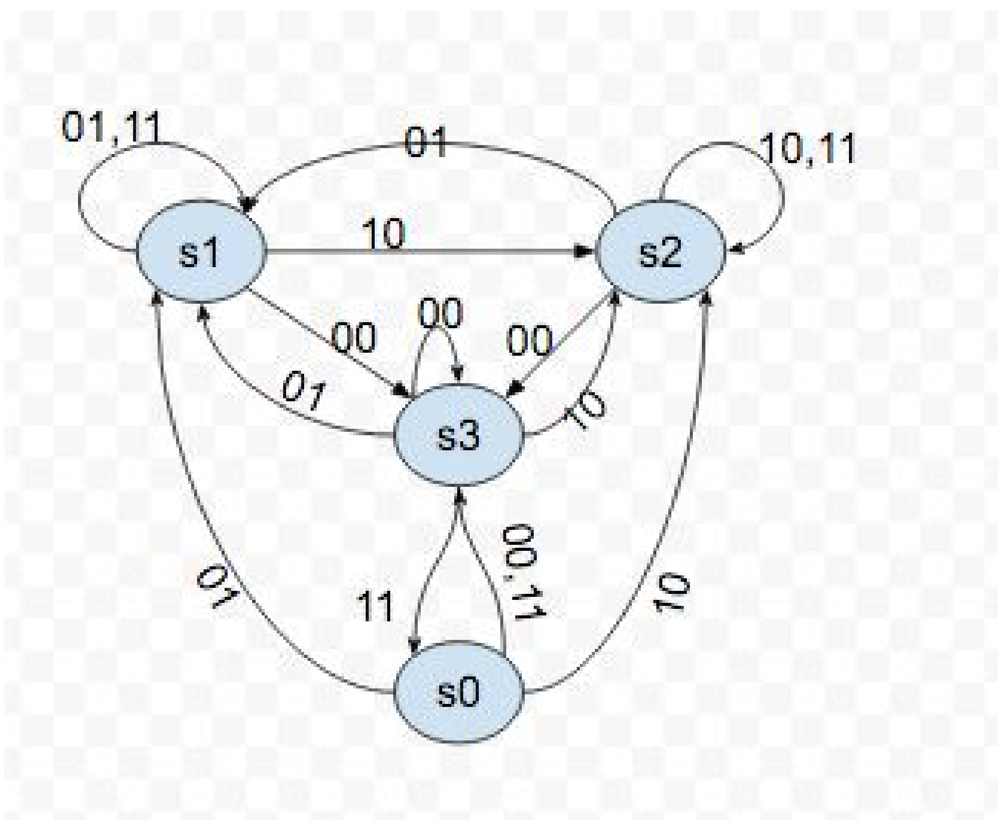
| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 6 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 7 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 8 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 10 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 11 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 12 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 13 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 14 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 15 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |

using S0 = 00, S1= 01, S2 = 10, S3=11

| id | current state | b | a | state (t1) | state (t2) | state (t3) |
|---|---|---|---|---|---|---|
| 0 | S0 | 0 | 0 | S3 | S3 | S3 |
| 1 | S0 | 0 | 1 | S3 | S1 | S1 |
| 2 | S0 | 1 | 0 | S3 | S2 | S2 |
| 3 | S0 | 1 | 1 | S3 | S0 | S3 |
| 4 | S1 | 0 | 0 | S3 | S3 | S3 |
| 5 | S1 | 0 | 1 | S1 | S1 | S1 |
| 6 | S1 | 1 | 0 | S3 | S2 | S2 |
| 7 | S1 | 1 | 1 | S1 | S1 | S1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 8 | S2 | 0 | 0 | S3 | S3 | S3 |
| 9 | S2 | 0 | 1 | S3 | S1 | S1 |
| 10 | S2 | 1 | 0 | S2 | S2 | S2 |
| 11 | S2 | 1 | 1 | S2 | S2 | S2 |
| 12 | S3 | 0 | 0 | S3 | S3 | S3 |
| 13 | S3 | 0 | 1 | S1 | S1 | S1 |
| 14 | S3 | 1 | 0 | S2 | S2 | S2 |
| 15 | S3 | 1 | 1 | S0 | S3 | S0 |

**state diagram:**
**You can see that the diagram has the same structure as the NOR SR latch from the lecture slide.**




**00 is the invalid input in this case, by replacing it with dont cares we will have just two state S1 and S2:**

| X(t)\ba | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | x | 0 | 0 | 1 |
| 1 | x | 0 | 1 | 1 |

**X (t+1) = a' + bX(t)**

**rubric:**

- **question 2: 18 pts**
  - **3pts each**
  - **give full credit to final answer**
  - **if final answer in wrong, give 2 points for good reasoning**
- **3 I . 1 10 pts for attempt**
- **3 I . 2 10 pts for attempt**
- **3 II . 1 10 pts for attempt**
- **3 || . 2 16 pts (-10 if its wrong)**
- **4 I . 1 8pts race condition in table**
- **4 I . 2 5pts for attempt**
- **4 I .3 5pts for attempt**
- **4 II. 1 8pts race condition in table**
- **4 || 2 5pts for attempt**
- **4 || 3 5pts for attempt**