Rubric

1 10 for attempt

2 10 for attempt

3 20 -2 for each incorrect term, deduct max -18

4.1 10 points for each equation. -3 for each incorrect/missing/extra terms in the equation

-5 for any extra equation.

4.2 10 for attempt

5.1 20 full credit if correct equation. -6 for each incorrect/missing/extra term in the equations.

-5 for any extra equation.

5.2 10 for attempt

1. Boolean Algebra

1. To verify if the system is a Boolean Algebra
   a. Identity element

      There exists identity element with respect to the operators.

      c is an identity element with respect to #. For each element p:

      p#c = c#p = p

      0 is an identity element with respect to &, such that for every element p:

      p&0 = 0&p = p

b. Commutativity

For all elements x,y ε M, the following property is satisfied:

x # y = y # x

x & y = y # x

eg. a # b = b # a = 0

a &b = b& a = c

This can be observed from the symmetric structure of the tables.

c. Distributivity

i. Operator # must be distributive over operator &. For elements p,q,r ε M, the following property must hold:

p#(q&r) = p#q & p#r

p	q	r	p#(q&r)	p#q & p#r
0	a	b	0	0
0	b	c	0	0
0	a	c	0	0
0	0	0	0	0
a	0	b	0	0
a	0	c	a	a
a	b	c	a	a
a	a	a	a	a
b	a	0	0	0
b	a	c	b	b
b	c	0	b	b
b	b	b	b	b
c	a	b	c	c
c	a	0	a	a
c	b	0	b	b
c	c	c	c	c

Note that this table only has 32 rows instead of 64, since the commutativity property has been proved above and hence:
x#(z&y) = x#(y&z)
So it is sufficient to prove only one of the above is distributive.

ii. Operator & must be distributive over operator #. For elements p,q,r ε M, the following property must hold:

p&(q#r) = p&q # p&r

p	q	r	p&(q#r)	p&q # p&r
0	a	b	0	0
0	b	c	b	b
0	a	c	a	a
0	0	0	0	0
a	0	b	a	a
a	0	c	a	a
a	b	c	c	c
a	a	a	a	a
b	a	0	b	b
b	a	c	c	c
b	c	0	b	b
b	b	b	b	b
c	a	b	c	c
c	a	0	c	c
c	b	0	c	c
c	c	c	c	c

Note that this table only has 32 rows instead of 64, since the commutativity property has been proved above and hence:
x&(z#y) = x&(y#z)
So it is sufficient to prove only one of the above is distributive.

The distributivity property holds for both operators as can be seen in the previous tables. Therefore the given system is distributive.

d. Complement
M is a multi-valued mathematical system. c and 0 are the identity elements with respect to operators # and & respectively. For each element x ε M, there exists an element x' ε M, such that:

x # x’ = 0
x & x’ = c

From the tables, there are two such pairs: (a,b) and (0,c) satisfying the above condition.
a#b = 0 and a&b = c
0#c = 0 and 0&c = c
Thus the complement exists.

As the given system satisfies the Huntington postulates as proved above, it is a Boolean Algebra.

2.

Element x	**Complement x’**
0	c
a	b
b	a
c	0

2. **Bit Counting Machine**

a	b	c	d	e	**s2**	**s1**	**s0**
0	0	0	0	0	**0**	**0**	**0**
0	0	0	0	1	**0**	**0**	**1**
0	0	0	1	0	**0**	**0**	**1**
0	0	0	1	1	**0**	**1**	**0**
0	0	1	0	0	**0**	**0**	**1**
0	0	1	0	1	**0**	**1**	**0**
0	0	1	1	0	**0**	**1**	**0**
0	0	1	1	1	**0**	**1**	**1**
0	1	0	0	0	**0**	**0**	**1**

0	1	0	0	1	**0**	**1**	**0**
0	1	0	1	0	**0**	**1**	**0**
0	1	0	1	1	**0**	**1**	**1**
0	1	1	0	0	**0**	**1**	**0**
0	1	1	0	1	**0**	**1**	**1**
0	1	1	1	0	**0**	**1**	**1**
0	1	1	1	1	**1**	**0**	**0**
1	0	0	0	0	**0**	**0**	**1**
1	0	0	0	1	**0**	**1**	**0**
1	0	0	1	0	**0**	**1**	**0**
1	0	0	1	1	**0**	**1**	**1**
1	0	1	0	0	**0**	**1**	**0**
1	0	1	0	1	**0**	**1**	**1**
1	0	1	1	0	**0**	**1**	**1**
1	0	1	1	1	**1**	**0**	**0**
1	1	0	0	0	**0**	**1**	**0**
1	1	0	0	1	**0**	**1**	**1**
1	1	0	1	0	**0**	**1**	**1**
1	1	0	1	1	**1**	**0**	**0**
1	1	1	0	0	**0**	**1**	**1**
1	1	1	0	1	**1**	**0**	**0**
1	1	1	1	0	**1**	**0**	**0**
1	1	1	1	1	**1**	**0**	**1**

s0(a,b,c,d,e) = Σ(1, 2, 4, 7, 8, 11, 13, 14, 16, 19, 21, 22, 25, 26, 28, 31)

s1(a,b,c,d,e) = Σ(3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 22, 24, 25, 26, 28)

s2(a,b,c,d,e) = Σ(15, 23, 27, 29, 30, 31)

3. **Priority Encoder**

a6	a5	a4	a3	a2	a1	a0	**d2**	**d1**	**d0**
0	0	0	0	0	0	X	0	0	0
X	X	X	X	X	1	0	0	0	1
X	X	X	X	1	0	0	0	1	0
X	X	X	1	0	0	0	0	1	1
X	X	1	0	0	0	0	1	0	0
X	1	0	0	0	0	0	1	0	1
1	0	0	0	0	0	0	1	1	0

d0 = a1.a0’ + a3.a2’.a1’.a0’ + a5.a4’.a3’.a2’.a1’.a0’
d1 = a2.a1’.a0’ + a3.a2’.a1’.a0’ + a6.a5’.a4’.a3’.a2’.a1’.a0’
d2 = a4.a3’.a2’.a1’.a0’ + a5.a4’.a3’.a2’.a1’.a0’ + a6.a5’.a4’.a3’.a2’.a1’.a0’

4.**Minimal Sum of Products Expression**

1.

a \ bc	00	01	11	10
0	0	1	1	0
1	1	0	1	X

Prime implicants: a’c, ac’, ab, bc
Essential prime implicants: a’c, ac’
f (a,b,c) = a’c + ac’ + ab
OR
f (a,b,c) = a’c + ac’ + bc

2.

cd \ ab	00	01	11	10
00	1	0	0	X
01	1	X	0	1
11	0	0	1	1
10	X	1	X	X

Prime implicants: cd’, ac, ab’, b’d’, a’c’d, b’c’
Essential prime implicants: ac, cd’

f(a,b,c,d) = ac + cd’ + b’c’

5. Minimal Product of Sums Expression

1.

a \ bc	00	01	11	10
0	0	1	X	1
1	X	0	1	0

Prime implicants: b+c, a’ + b, a’ + c
Essential prime implicants: b+c, a’+b, a’ + c
f(a,b,c) = (b+c).(a’+b).(a’+c)

2.

cd / ab	00	01	11	10
00	0	0	1	0
01	0	X	X	0
11	1	X	0	1
10	X	1	0	0

Prime implicants: a+ c, c+d’, b+c, b+d, b’+d’, a’+b’+c’, a’+c’+d

Essential prime implicants: a+c

f(a,b,c,d) = (a+c). (b’+d’).(a’+c’+d).(b+c)

OR

f(a,b,c,d) = (a+c). (a’+b’+c’).(c+d’).(b+d)