Homework#1 solution

Grading Rubric:

2.1.A 10pts for the attempt
2.1.B 10pts for the attempt
2.2.A 10pts for the attempt
2.2.B 20pts
10pts for using Shannon's expansion in correct form,
5pts for simplifying each of the two terms
3.A 10pts for the attempt
3.B 10pts for the attempt
4.i 20pts deduct 1 point for each wrong value in the table, don't deduct more than 20 points
4.ii 5pts grade based on their given truth table, deduct 1 point for each extra/missing minterm
4.iii 5pts grade based on their given truth table, deduct 1 point for each extra/missing maxterm
-- if minterms and maxterms are used instead of each other given half credit
-- if the expanded form is not written give half credit.

2.1 Boolean Algebra Simplification

A. bd + acd' + abc = bd + acd'

L.H.S. = bd + acd' + abc
= bd + acd' + abc.1 (identity)
= bd + acd' + abc(d+d') (complements)
= bd + acd' + abcd + abcd' (distributive)
= bd + abcd + acd' + abcd' (commutative)
= bd (1 + ac) + acd' (1 + b) (distributive)
= bd + acd'

L.H.S = R.H.S

B. (b + d)(a + c + d')(a + b + c) = (b + d)(a + c + d')
L.H.S. = (b + d)(a + c + d')(a + b + c)
= (b + d)(a + c + d')(a + b + c + 0) (identity)
= (b + d)(a + c + d')(a + b + c + d.d') (complements)
= (b + d)(a + c + d')(a + b + c + d)(a + b + c + d') (distributive)
= (b + d)(a + b + c + d)(a + c + d')(a + b + c + d') (commutative)
= (b + d)(1 + a + c)(a + c + d')(1 + b) (distributive)
= (b + d)(a + c + d')

L.H.S. = R.H.S

2.2 Using Shannnon's Expansion

A. bd + acd' + abc = bd + acd'

F(a,b,c,d) = bd + acd' + abc

F(a,b,c,d) = d. F(a,b,c,1) + d'.F(a,b,c,0)

F(a,b,c,1) = b.1 + a.c.0 + a.b.c
= b + abc
= b(1 + ac)
= b

$F(a,b,c,0) = b.0 + a.c.1 + a.b.c$
$= ac + abc$
$= ac(1 + b)$
$= ac$

$F(a,b,c,d) = d.F(a,b,c,1) + d'.F(a,b,c,0)$
$= d.b + d'.ac$
$= bd + acd'$

B. $(b + d)(a + c + d')(a + b + c) = (b + d)(a + c + d')$

$F(a,b,c,d) = (b + d)(a + c + d')(a + b + c)$

$F(a,b,c,d) = (d + F(a,b,c,0))(d' + F(a,b,c,1))$

$F(a,b,c,0) = (b + 0)(a+ c + 1)(a+ b + c)$
$= (b).(a + b + c)$
$= b(1 + a + c)$
$= b$

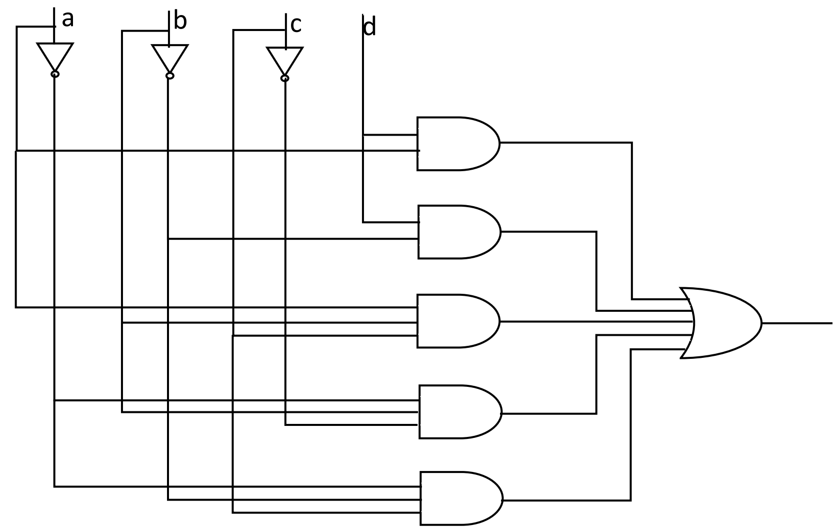
$F(a,b,c,1) = (b + 1)(a+c+0)(a+ b+c)$
$= (a + c)(a + b + c)$
$= (a+c)(1+ b)$
$= a+ c$

$F(a,b,c,d) = (d + F(a,b,c,0))(d' + F(a,b,c,1))$
$= (d + b)(d' + a + c)$
$= (b+ d)(a+ c+ d')$

3.
A. i) $a'bc' + a'b'd + a'b'cd' + bc'd + acd + abcd' + ac'd$
$= a'bc' + a'b'd + a'b'cd' + (a+a')bc'd + acd + abcd' + ac'd$ (identity)
$= a'bc' + a'b'd + a'b'cd + a'bc'd + abc'd + acd + abcd' + ac'd$ (distributive)
$= a'bc'(1 + d) + a'b'd + a'b'cd' + abc'd + abcd' + ad(c+c')$ (distributive)
$= a'bc' + a'b'd + a'b'cd' + abc'd + abcd' + ad$
$= a'bc' + a'b'd + a'b'cd' + a'b'c + ad(bc' + 1) + abcd'$ (consensus) // $(a'b'd + a'b'cd' = a'b'd + a'b'cd' + a'b'c)$
$= a'bc' + a'b'd + a'b'c(d'+1) + ad + abcd'$
$= a'bc' + a'b'd + a'b'c + ad + abcd' + abc$ (consensus) // $ad + abcd' = ad + abcd' + abc$
$= a'bc' + a'b'd + a'b'c + ad + abc(d'+1)$
$= a'bc' + a'b'd + a'b'c + ad + abc$
$= ad + a'b'd + a'bc' + a'b'c + abc$
$= ad + a'b'd + b'd + a'bc' + a'b'c + abc$ (consensus)
$= ad + b'd(a'+1) + a'bc' + a'b'c + abc$
$= ad + b'd + a'bc' + a'b'c + abc$

Other possible solution: $ad + c'd + abc + a'bc' + a'b'c$ and $b'd + c'd + abc + a'bc' + a'b'c$

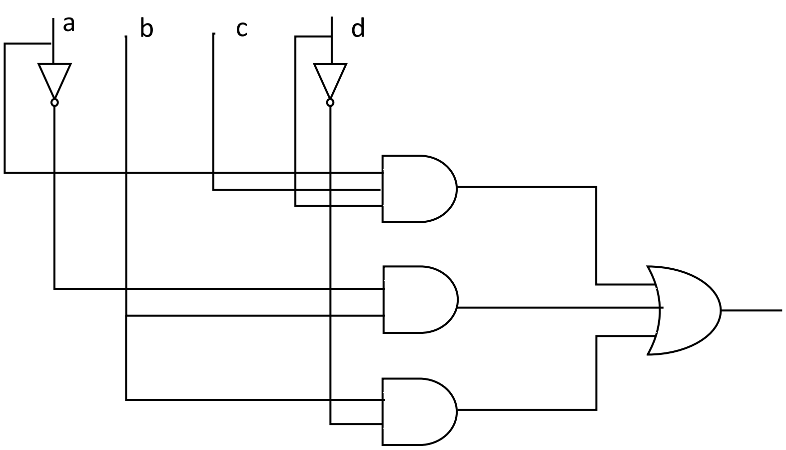
ii).



iii). Literals :- 7 (a,a',b,b',c,c',d) Operators:- 17 operators (3 NOT, 5 AND, 1 OR)
Gates: - 9 gates, 13 nets, 30 pins

B. i) (a + b + c + d')(a + b + c + d)(a + b + c')(a' + b + c + d)(a' + c + d')(a' + b + c' + d)
= (a + b + c)(a + b + c')(a' + b + d)(a' + c + d') (distributive)
= (a + b)(a' + b + d)(a' + c + d')
= (a+b)(a'+ b +d) (b +d) (a' +c +d') (consensus) //(a+b)(a'+b+d) = (a+b)(a'+b+d)(b+d)
= (a+b)(b + d) (a' +1) (a' +c +d') (distributive)
= (a+b)(b + d)(a' +c +d')
= (ad +b) (a' +c +d')
= acd + a'b + bc + bd'
= acd + a'b + bc(d +d') + bd'
= acd + a'b + bcd + bcd' + bd'
= acd + a'b + bd'(c+1)
= acd + a'b + bd'

ii).



iii).
Literals :- 6 (a,a',b,c,d,d') Operators:- 8 operators (2 NOT, 3 AND, 1 OR)
Gates: - 6 gates, 10 nets, 18 pins

4. i.

X4	X3	X2	X1	X0	**Y**
0	0	0	0	0	**0**
0	0	0	0	1	**0**
0	0	0	1	0	**0**
0	0	0	1	1	**0**
0	0	1	0	0	**0**
0	0	1	0	1	**0**
0	0	1	1	0	**0**
0	0	1	1	1	**1**
0	1	0	0	0	**0**
0	1	0	0	1	**0**
0	1	0	1	0	**0**
0	1	0	1	1	**1**
0	1	1	0	0	**0**
0	1	1	0	1	**1**
0	1	1	1	0	**1**
0	1	1	1	1	**1**
1	0	0	0	0	**0**
1	0	0	0	1	**0**
1	0	0	1	0	**0**
1	0	0	1	1	**1**
1	0	1	0	0	**0**
1	0	1	0	1	**1**
1	0	1	1	0	**1**
1	0	1	1	1	**1**
1	1	0	0	0	**0**
1	1	0	0	1	**1**
1	1	0	1	0	**1**
1	1	0	1	1	**1**
1	1	1	0	0	**1**
1	1	1	0	1	**1**
1	1	1	1	0	**1**
1	1	1	1	1	**1**

ii). F(x4,x3,x2,x1,x0) = x4’.x3’x2.x1.x0 + x4’.x3.x2’.x1.x0 + x4’.x3.x2.x1’.x0 + x4’.x3.x2.x1.x0’ + x4’.x3.x2.x1.x0 + x4.x3’.x2’.x1.x0 + x4.x3’.x2.x1’.x0 + x4.x3’.x2.x1.x0’ + x4.x3’.x2.x1.x0 + x4.x3.x2’.x1’.x0 + x4.x3.x2’.x1.x0’ + x4.x3.x2’.x1.x0 + x4.x3.x2.x1’.x0’ + x4.x3.x2.x1’.x0 + x4.x3.x2.x1.x0’ + x4.x3.x2.x1.x0

F(x4,x3,x2,x1,x0) = Σm(7,11,13,14,15,19,21,22,23,25,26,27,28,29,30,31)

iii). F(x4,x3,x2,x1,x0) = (x4 + x3 + x2 + x1 + x0).(x4 + x3 + x2 + x1 + x0’).(x4 + x3 + x2 + x1’ + x0).(x4 + x3 + x2 + x1’ + x0’).(x4 + x3 + x2’ + x1 + x0).(x4 + x3 + x2’ + x1 + x0’).(x4 + x3 + x2’ + x1’ + x0).(x4 + x3’ + x2 + x1 + x0).(x4 + x3’ + x2 + x1 + x0’).(x4 + x3’ + x2 + x1’ + x0).(x4 + x3’ + x2’ + x1 + x0).(x4’ + x3 + x2 + x1 + x0).(x4’ + x3 + x2 + x1 + x0’).(x4’ + x3 + x2 + x1’ + x0).(x4’ + x3 + x2’ + x1 + x0).(x4’ + x3’ + x2 + x1 + x0)

F(x4,x3,x2,x1,x0) = ΠM(0,1,2,3,4,5,6,8,9,10,12,16,17,18,20,24)