Midterm2 Solution

1 Universal Set of Gates:

1.1 Define a universal set of gates (8 points).

Universal set is a set of gates such that every Boolean function can be implemented with the gates in the set. In other words, if the set of gates can implement AND, OR, and NOT gates, the set is universal.

1.2 Check if the set in the following list is universal and explain your decision. Assuming constants 0 and 1 are available as inputs (12 points).

1.2.1 {OR, NOT}
Universal
NOT : Given
OR : Given
AND : (A'+B')' = AB = A AND B

1.2.2 {NAND}
Universal
NOT : 1 NAND A = A' = NOT A
AND : (A NAND B)' = A AND B
OR : OR gate can be constructed with NOT and AND gates since A+B = (A'B')'

1.2.3 {$f(x, y)$}, where $f(x, y) = x'y'$
Universal
NOT : f(A, 0) = A'·0' = A'·1 = A' = NOT A
OR : (f(A, B))' = (A'B')' = A + B = A OR B
AND : AND gate can be constructed with NOT and OR gates since A·B = (A'+B')'

1.2.4 {$f(x, y, z)$}, where $f(x, y, z) = xyz' + xy'z + x'yz$
Universal
NOT : f(A, 1, 1) = A·1·1' + A·1'·1 + A'·1·1 = A·1·0 + A·0·1 + A'·1·1 = A' = NOT A
AND : f(A, B, 0) = A·B·0' + A·B'·0 + A'·B·0 = A·B·1 + 0 + 0 = A AND B
OR : OR gate can be constructed with NOT and AND gates since A+B = (A'B')'

Rubric

- If the answers are correct (i.e., universal for all questions), then 12 points.
- If the answers are incorrect, then see the explanation. If the explanation is correct, then +1 point.

2 Other Types of Gates: Consider the function f(x, y) where ⊕ is an Exclusive OR operator:

$$f(x, y) = (x + y') \oplus (x'y) \oplus (x' + y) \oplus x \oplus y'$$

2.1 Evaluate $f(1, y)$ and $f(0, y)$. (6 points)

$f(1, y)$	$= (1 + y') \oplus (1'y) \oplus (1' + y) \oplus 1 \oplus y'$	
	$= 1 \oplus 0 \oplus y \oplus 1 \oplus y'$	(by 1+A=1, 0+A=A, 0·A=0)
	$= 1 \oplus 0 \oplus 1 \oplus y \oplus y'$	(by commutative law)
	$= 0 \oplus y \oplus y'$	(by definition of XOR)
	$= 0 \oplus 1$	(by definition of XOR)
	$= 1$	(by definition of XOR)

$f(0, y)$	$= (0 + y') \oplus (0'y) \oplus (0' + y) \oplus 0 \oplus y'$	
	$= y' \oplus y \oplus 1 \oplus 0 \oplus y'$	(by 0+A=A, 1·A=A, 1+A=1)
	$= 1 \oplus 1 \oplus y'$	(by definition of XOR)
	$= 0 \oplus y'$	(by definition of XOR)
	$= y'$	(by 0⊕A=A)

Rubric

- 3 points for $f(1, y)$
  If the answer is not simplified, -1 point
  If the answer is not correct, -2 points
- 3 points for $f(0, y)$
  If the answer is not simplified, -1 point
  If the answer is not correct, -2 points

2.2 Simplify $f(x, y)$ to minimal sum of products expression. No restriction on the method. However, you must explain your approach. (9 points)

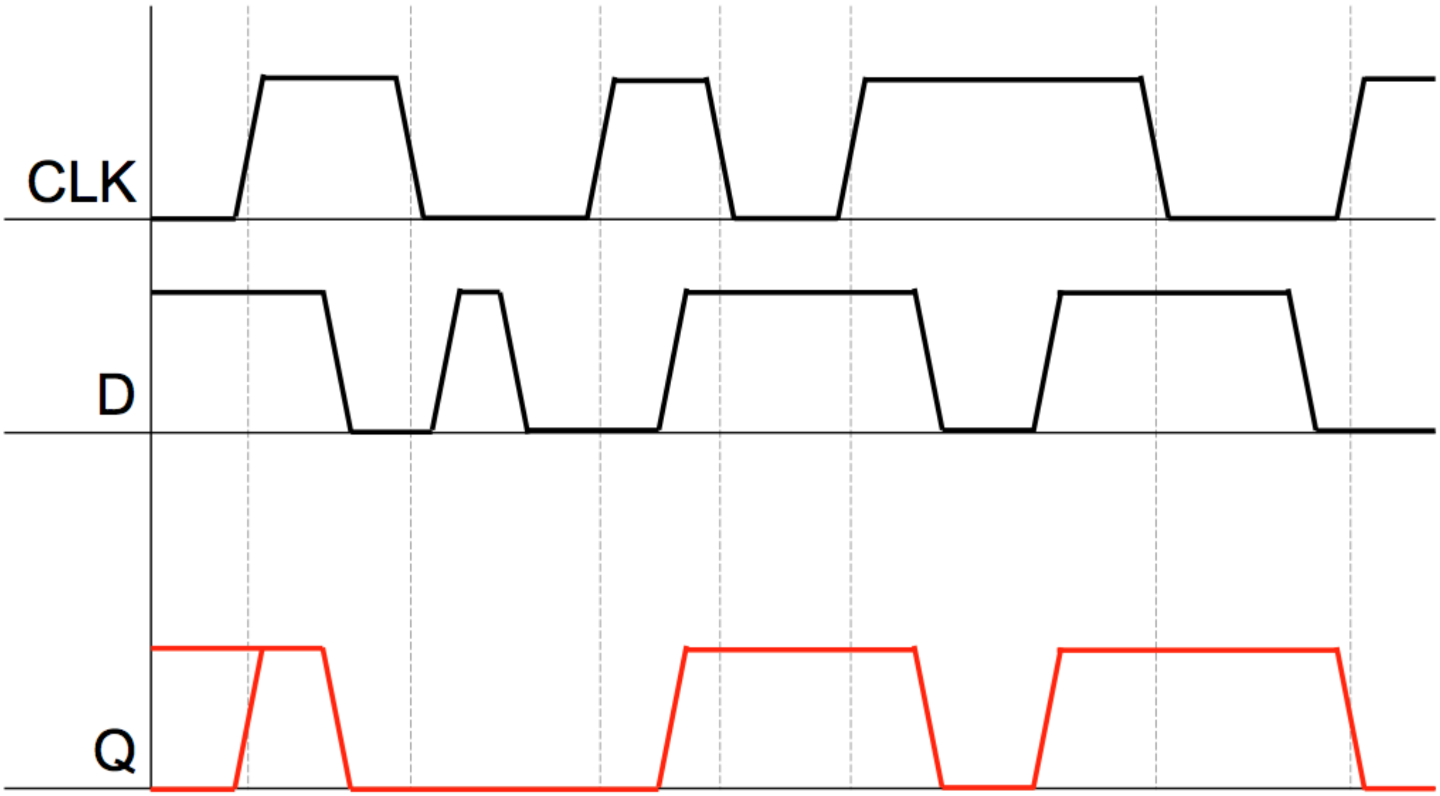
$f(x, y)$	$= x \cdot f(1, y) + x' \cdot f(0, y)$	4 points
	$= x \cdot 1 + x'y'$	3 points
	$= x + y'$	2 points

Rubric

- If everything is correct: 9 points
- If the answer is not simplified: -2 points
- If the answer is incorrect because of the previous problem: give 4 points total.

3 Timing Diagram of Latch and Flip-Flop:

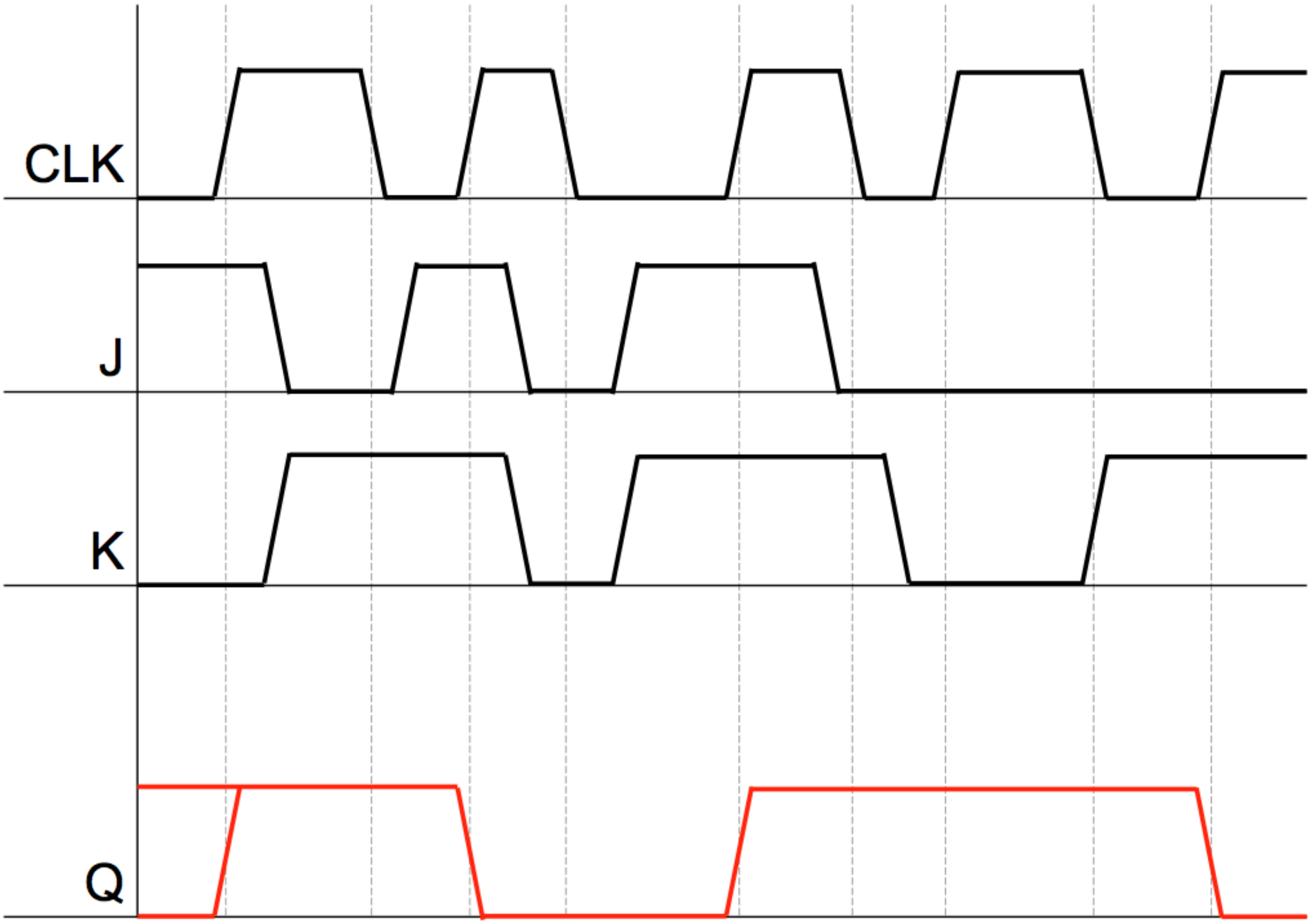
3.1 Given the input waveforms shown below, sketch the output, Q, of a D latch (10 points).



Rubric

- Each clock cycle: 2 points
- If connections between clock cycles are correct, + 2 points

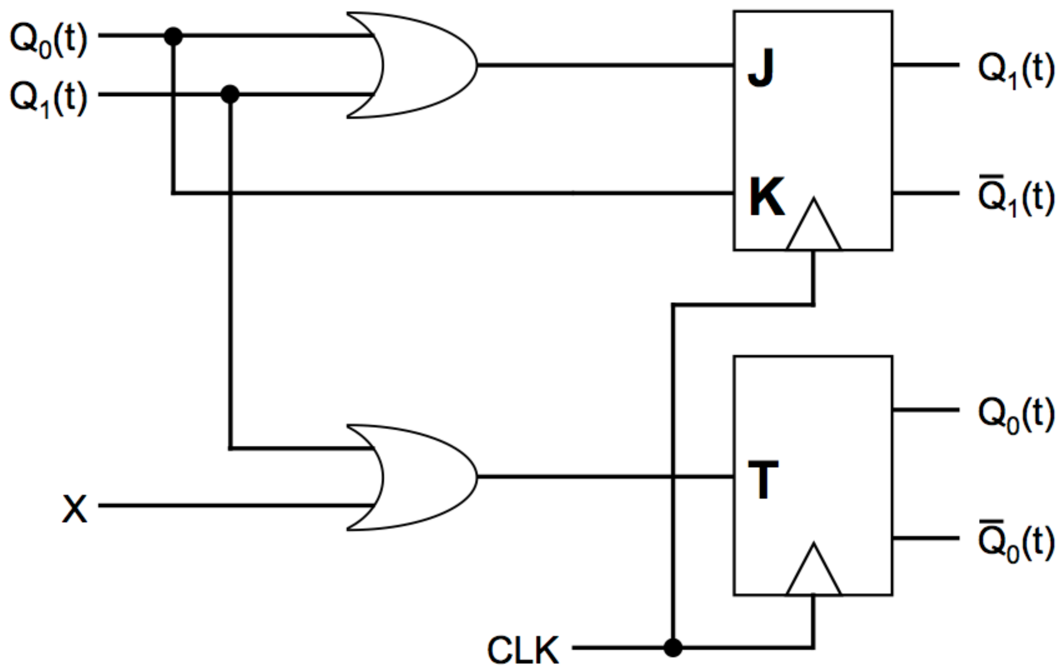
3.2 Given the input waveforms shown below, sketch the output, Q, of a JK flop-flop. The flip-flop is triggered at the rising edge of the clock. (10 points).



Rubric

- Each clock cycle: 2 points

4 (Finite State Machine Specification) Consider the following circuit.



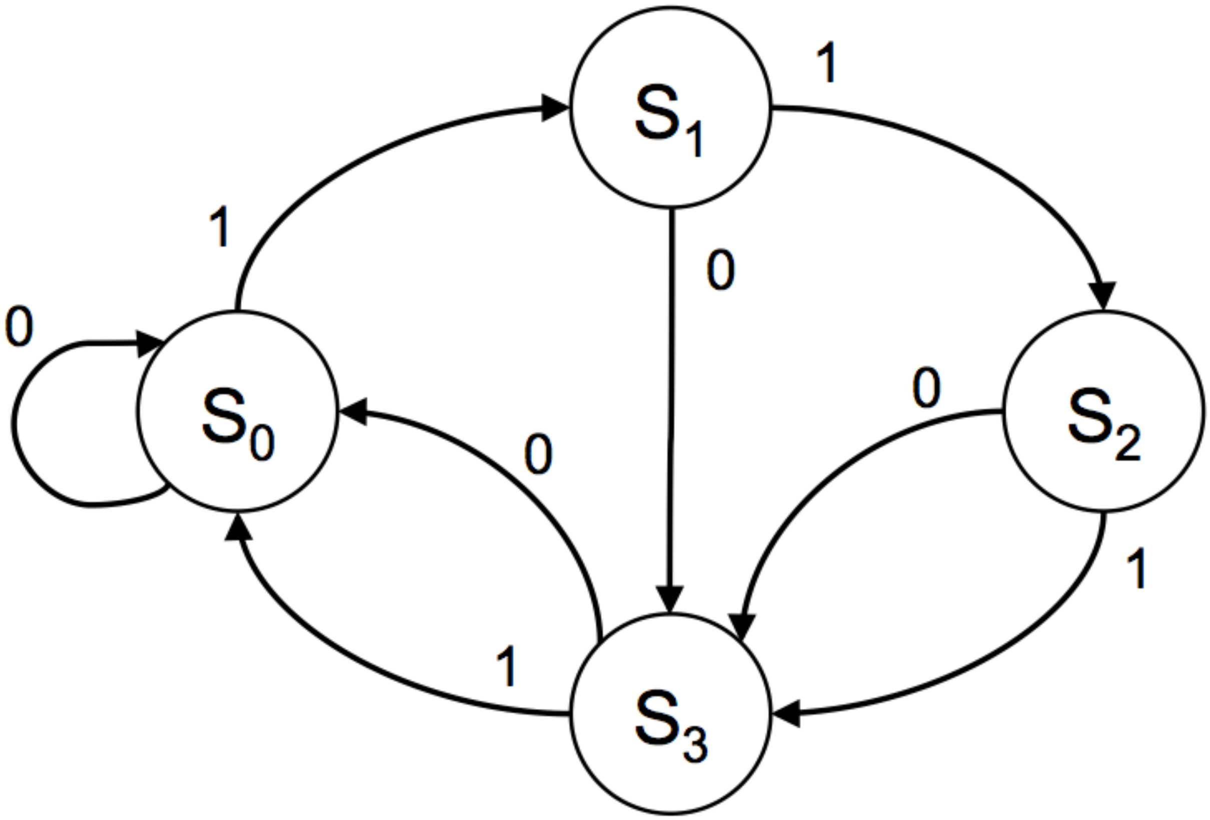
4.1 Write the state transition table (10 points).

Current State	$Q_1(t)$	$Q_0(t)$	X	J_1	K_1	T	$Q_1(t+1)$	$Q_0(t+1)$	Next State
S_0	0	0	0	0	0	0	0	0	S_0
	0	0	1	0	0	1	0	1	S_1
S_1	0	1	0	1	1	0	1	1	S_3
	0	1	1	1	1	1	1	0	S_2
S_2	1	0	0	1	0	1	1	1	S_3
	1	0	1	1	0	1	1	1	S_3
S_3	1	1	0	1	1	1	0	0	S_0
	1	1	1	1	1	1	0	0	S_0

Rubric

- If everything correct, 10 points
- $Q_1(t+1)$: 5 points
  Each row has 0.5 point.
  Trial 1 point.
- $Q_0(t+1)$: 5 points
  Each row has 0.5 point.
  Trial 1 point.

4.2 Sketch the state diagram (10 points).



Rubric

- Trial: 2 points
- Draw four states: 2 points
- Each arrow with input has 0.5 point
- Everything correct: +2 points
- Note: Outputs in student's answer do not affect their scores.

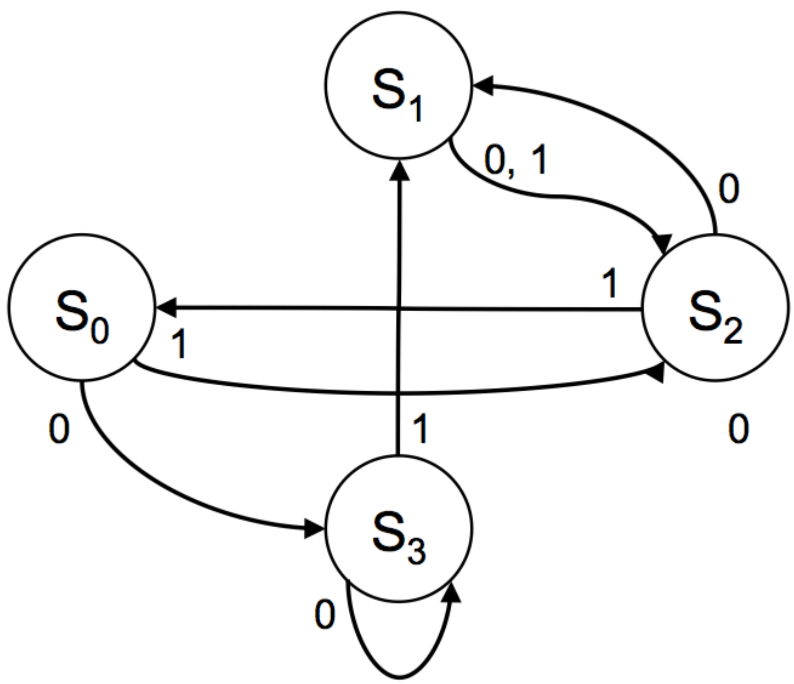
5 A state machine has one input $x(t)$ and two-bit state ($Q_1(t)$, $Q_0(t)$). The machine is described by the following state equations.

$$Q_1(t+1) = Q_1'(t) + x'(t)Q_0(t),$$

$$Q_0(t+1) = Q_1(t)Q_0(t) + x'(t)Q_0'(t).$$

5.1 Write the state transition table and draw the state diagram (10 points).

Current State	$Q_1(t)$	$Q_0(t)$	$x(t)$	$Q_1(t+1)$	$Q_0(t+1)$	Next State
S_0	0	0	0	1	1	S_3
	0	0	1	1	0	S_2
S_1	0	1	0	1	0	S_2
	0	1	1	1	0	S_2
S_2	1	0	0	0	1	S_1
	1	0	1	0	0	S_0
S_3	1	1	0	1	1	S_3
	1	1	1	0	1	S_1



Rubric

- Table: 6 points
  Q1(t+1): 3 points
  Incorrect row -0.3 point
  Q0(t+1): 3 points
  Incorrect row -0.3 point
- Diagram: 4 points
  Each state with two correctly drawn arrows: 1 point
- Note: Outputs in student's answer do not affect their scores.

5.2 Use two JK flop-flops and a minimal AND-OR-NOT network to implement he machine. Show your derivation (K-maps) and draw the logic diagram (15 points).

Current State	$Q_1(t)$	$Q_0(t)$	$x(t)$	$Q_1(t+1)$	$Q_0(t+1)$	Next State	J_1	K_1	J_0	K_0
S_0	0	0	0	1	1	S_3	1	X	1	X
	0	0	1	1	0	S_2	1	X	0	X
S_1	0	1	0	1	0	S_2	1	X	X	1
	0	1	1	1	0	S_2	1	X	X	1
S_2	1	0	0	0	1	S_1	X	1	1	X
	1	0	1	0	0	S_0	X	1	0	X
S_3	1	1	0	1	1	S_3	X	0	X	0
	1	1	1	0	1	S_1	X	1	X	0

$X \backslash Q_1(t)Q_0(t)$	00	01	11	10
0	1	1	X	X
1	1	1	X	X

$J_1=1$

$X \backslash Q_1(t)Q_0(t)$	00	01	11	10
0	X	X	0	1
1	X	X	1	1

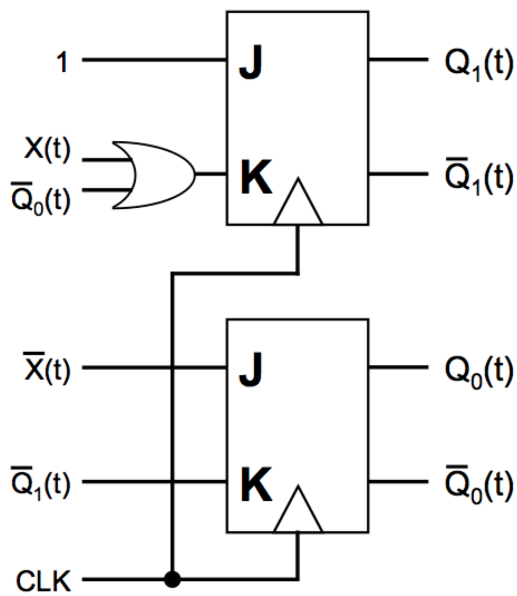
$K_1=X(t) + Q_0'(t)$

$X \backslash Q_1(t)Q_0(t)$	00	01	11	10
0	1	X	X	1
1	0	X	X	0

$J_0=X'(t)$

$X \backslash Q_1(t)Q_0(t)$	00	01	11	10
0	X	1	0	X
1	X	1	0	X

$K_0=Q_1'(t)$



Rubric

- JK Values: 5 points
  Trial: 1 point
  Each correct value (i.e., column in the table): 1 point
- K-maps: 5 points
  Trial: 1 point
  Each correct K-map: 1 point
- Logic diagram: 5 points
  Trial: 1 point
  Each correct JK flip-flop: 2 points