Midterm 2 Solution

1
1.1
Universal set of gates is a set of gates such that every Boolean function can be implemented with gates in this set.
Alter: A Universal set of gates is one from which the three basic Boolean functions (NOT, AND and OR) can be implemented.

1.2
1.2.1 {OR, NOT}
Yes. AND can be implemented as $ab = (a' + b')'$

1.2.2 {NAND, NOR}
Yes.
NOT can be implemented by tying one input of NAND gate to 1.
AND can be implemented by inverting the output of NAND gate i.e. $ab = ((ab)')'$
Similarly OR can implemented by inverting the output of a NOR gate

1.2.3 $\{f(x, y)\}$, where $f(x, y) = x'y$
Yes.
$f(x, 1) = x'$ i.e. NOT can be implemented
$f(x', y) = xy$ i.e AND can be implemented
OR can be implemented using the above two because $x + y = (x'y')'$

1.2.4 $\{f(x, y, z)\}$, where $f(x, y, z) = (x'y' + xy)z$
Yes.
f(x,0,1)=x' i.e. NOT can be implemented
f(x,1,z)=xz i.e AND can be implemented
OR can be implemented using the above two because x+y=(x'y')'

Rubric:

- For each of the above parts, award 3 points if the conclusion as well as the reason is correct.
- For parts 1.2.1 and 1.2.2, deduct 1 point if the conclusion is correct but reason is not provided.
- For parts 1.2.3 and 1.2.4, deduct 2 points if the conclusion is correct bu the reason is not provided.
- If the conclusion is incorrect, award 1 point for every functionality that is shown to be implemented by that set

2
If $AB = 0$ then $A \bigoplus B = A + B$.
Proof:
A = A(B+B') (Complement Law)
= AB+AB' (Distributivity)
= 0+AB' ($AB = 0$)
Also,
B = B(A+A') (Complement Law)
= AB+A'B (Distributivity)
= 0+A'B ($AB = 0$)

Therefore, A+B=AB'+A'B

Alter:
Let $X = A \bigoplus B$ and $Y = A + B$
$X = Y$ iff $X + Y' = 1$ and $XY' = 0$
X+Y' = A $\bigoplus B + (A + B)'$
$= AB' + A'B + A'B' \qquad (DeMorgan's)$
$= AB' + A'B + A'B' + A'B' \qquad$ (A+A=A)
$= B'(A + A') + A'(B + B') \qquad (Distributivity)$
$= B' + A' \qquad (Identity)$
$= (AB)' \qquad (DeMorgan's)$
$= 1 \qquad$ (AB=0)

XY' = (A $\bigoplus B)(A + B)'$
$= (AB' + A'B)(A'B') \qquad (DeMorgan's)$
$= AB'A'B' + A'BA'B' \qquad (Distributivity)$
$= 0 \qquad (Complement)$

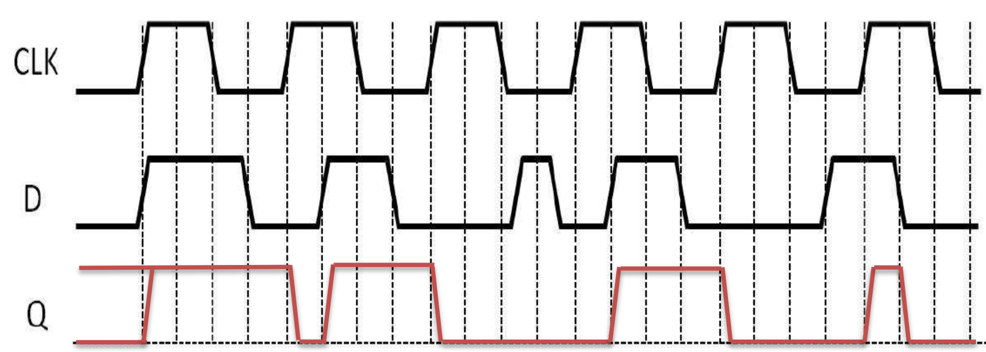
Rubric:

- Deduct 1 point if the laws/properties are not listed

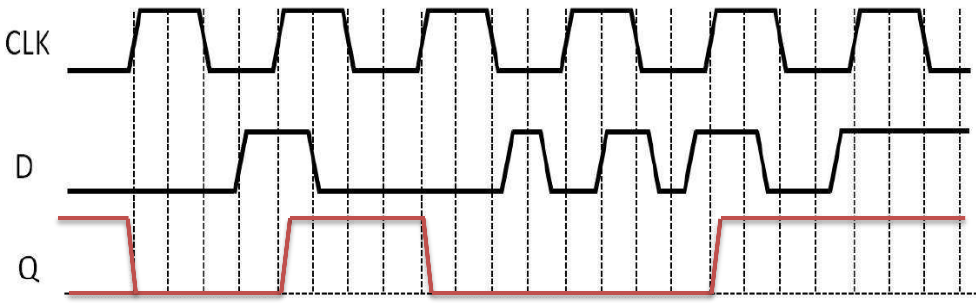
- For the alternative approach, award 5 points if the criterion for equality is listed, 5 points if $X + Y' = 1$ is proved correctly and 5 points if $XY' = 0$ is proved correctly

3
Rubric:
Only the outputs for the first 10 clock levels (includes both high and low levels) are considered. 1 point is deducted for every output at that level found incorrect.

3.1



3.2



4

4.1 Boolean expressions for the next states and the output.

$$
\begin{aligned}
Q_0(t+1) &= XQ_1(t) + XQ_0(t)' \\
Q_1(t+1) &= XQ_1(t) + XQ_0(t) \\
M &= X'Q_1(t)Q_0(t)
\end{aligned}
$$

According to those Boolean equations, the following transition table can be derived.

present state			input	next state			output
state	$Q_1(t)$	$Q_0(t)$	X	state	$Q_1(t+1)$	$Q_0(t+1)$	M
S_0	0	0	0	S_0	0	0	0
S_0	0	0	1	S_1	0	1	0
S_1	0	1	0	S_0	0	0	0
S_1	0	1	1	S_2	1	0	0
S_2	1	0	0	S_0	0	0	0
S_2	1	0	1	S_3	1	1	0
S_3	1	1	0	S_0	0	0	1
S_3	1	1	1	S_3	1	1	0

4.2

The following state diagram can be obtained from the transition table.

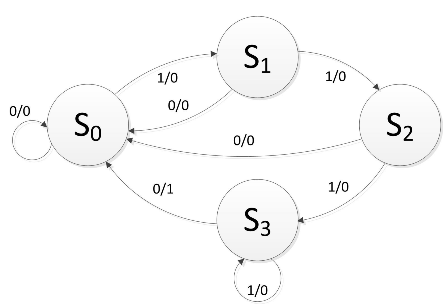


Figure 1: State diagram of Problem 4.

4.3

This FSM outputs $M = 1$ once finds pattern $X = 1110$.
Rubric:

cycle	1	2	3	4	5	6	7	8	9	10
X	1	1	1	0	0	0	1	1	1	0
state	S_0	S_1	S_2	S_3	S_0	S_0	S_0	S_1	S_2	S_3
M	0	0	0	1	0	0	0	0	0	1

- Get 7 points for correct transition table.
- Get 2 points for each correct Boolean function of Q_0, Q_1 and M if the transition table is incorrect.
- Deduct 2 points for each incorrect Boolean function but correct transition table.
- Deduct 1 points for one incorrect row in the transition table.
- Deduct 1 points for one incorrect transition in the state diagram.
- Deduct 1 points for one incorrect state/M in 4.3
- Deduct 2 points for correctly describing state diagram and the table in 4.3 of the incorrect transition table in 4.1.
- Deduct 7 points for incorrect state diagram based on incorrect transition table in 4.1.
- Deduct 7 points for no state diagram.
- Deduct 5 points for incorrect table in 4.3 based on incorrect transition table in 4.1.
- Deduct 3 points for incorrect label in the state diagram, e.g., no output on transition edges.
- Get 5 points for correct description of FSM functionality.
- Get 5 points for correct table in 4.3.
- Get 5 points for correct description of FSM functionality but with 1 cycle shift of M.
- Deduct 2 points for incorrect states in the table but correct description of FSM functionality.

V

T	$Q(t)$	$Q(t+1)$	D
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

In this table, T, $Q(t)$ and $Q(t+1)$ show the expected functionality of a T flip-flop, and D shows the corresponding values required for $Q(t+1)$. Boolean expression for D of a D flip-flop is

$$D = Q(t)'T + Q(t)T'.$$

The final logic diagram is as

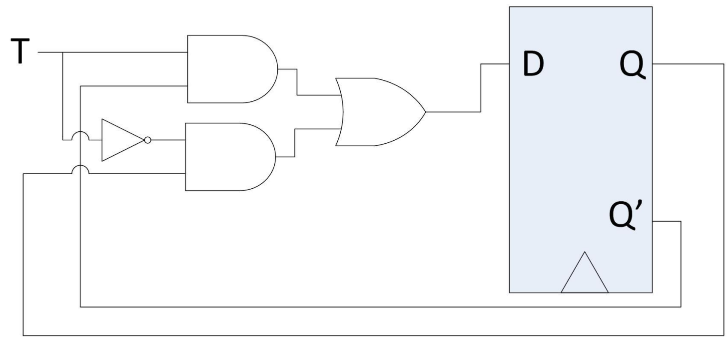


Figure 2: Logic diagram of Problem 5.

Rubric:

- Get 12 points for correct excitation table of $T(t)$, $Q(t)$, $Q(t+1)$ and $D(t)$.
- Get 15 points for implementing D flip-flop with T flip-flop.
- Get 2 points for correct K map from the incorrect excitation table.
- Get 6 points for incomplete Boolean expression for D.
- Get 12 points for correct Boolean expression for D.
- Deduct 2 points for minor notation incorrectness in the correct logic diagram.