CSE140: Midterm 1 Solution and Rubric

April 23, 2014

1 Short Answers

1.1 True or False (6pts)

1. A maxterm must include all input variables **(1pt)**
   True

2. A canonical product of sums is a product of minterms **(1pt)**
   False

3. To reduce an incompletely specified function to a sum of products form, the don't cares are always assumed to be zero for best results **(1pt)**
   False

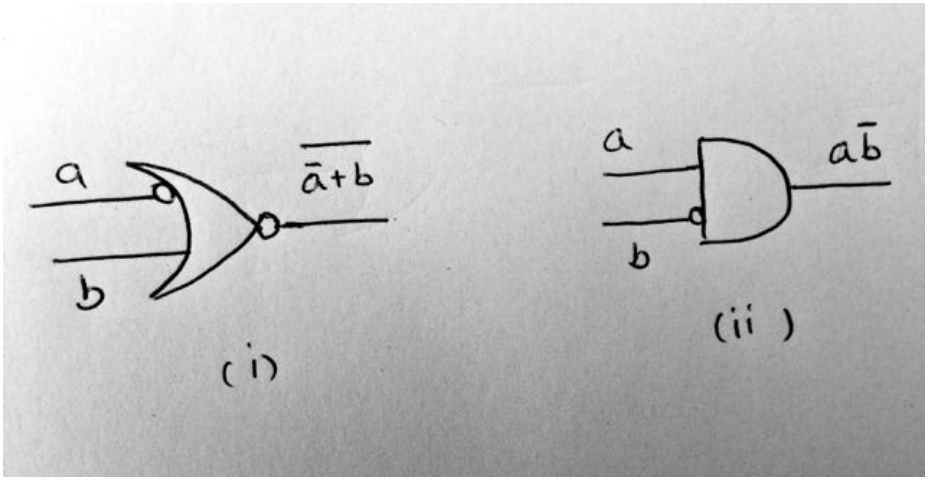
4. Pushing a bubble from the output of a logic gate towards the inputs always transforms the gate **(1pt)**
   True or **False**: Both are acceptable

5. In a K-map, non-essential primes are a proper subset of essential primes **(1pt)**
   False

6. In a K-map, two adjacent cells can sometimes differ in more than one literal?**(1pt)**
   False

1.2 Bubble Pushing

Draw two equivalent but non-identical circuits for the boolean expression $\overline{\bar{a} + b}$, **(4pts)**



Above circuit can be implemented with NOT gates instead of bubbles.
Rubric:
Each circuit 2 points.
Deduct 1 point for not showing input variables (maximum of 2 points)

2 Problem to Canonical Expression

A vending machine has four binary inputs (a, b, c, d) which correspond to four coin slots. The first slot is for a quarter and the rest of the three slots are for three dimes, which corresponds to one dime for each slot. Input $a = 1$ if the user deposited a quarter at the first slot otherwise $a = 0$. Likewise, each of the three inputs (b, c, d) indicate if the user deposited a dime in each of the last three slots. For example, if the input $(a, b, c, d) = (1, 1, 1, 0)$, then the user has deposited one quarter and two dimes, a total of $45 cents$. The output function $f = 1$ if the user has deposited coins worth more than or equal to $30 cents$, otherwise $f = 0$.

I. Write the truth table of the output f as a function of inputs a, b, c, d. **(5pts)**

id	a	b	c	d	f
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

Table 1: Truth table

Rubric:
Deduct 0.5 point for each incorrect row in the truth table (maximum deduction 5 pts)

II. Describe the function f in the canonical product-of-sums format. **(5pts)**

$f(a, b, c, d) = \prod M(0, 1, 2, 3, 4, 5, 6, 8)$

Rubric: Deduct 2 points if POS expression matches truth table, but table is wrong. Deduct 1 point if SOP instead of POS

3 Consensus Theorem

Prove the following equality using Boolean algebra. **(15pts)**

$a'c + a'b'd + b'c'd \;=\; a'c + b'c'd$

LHS: $a'c + a'b'd + b'c'd$

$= a'c + a'b'd(c + c') + b'c'd$

$= a'c + a'b'dc + a'b'dc' + b'c'd$ *Distributive Law*

$= a'c(1 + b'd) + b'dc'(a' + 1)$ *Commutative and Distributive Laws*

$= a'c(1) + b'c'd(1)$

$= a'c + b'c'd$

$=$ RHS

Rubric:
Deduct 2 points for each incorrect steps (max deduction 15 pts).
Deduct 5 points if Boolean Algebra theorems are not used for the proof (e.g. if an enumerative method is used)
No points deducted for using Shannon expansion

4 Shannon's Expansion

Prove the following equality using Shannon's expansion. **(15pts)**

$(a' + c)(a' + b' + d)(b' + c' + d) = (a' + c)(b' + c' + d)$

Apply Shannon's Expansion twice to LHS

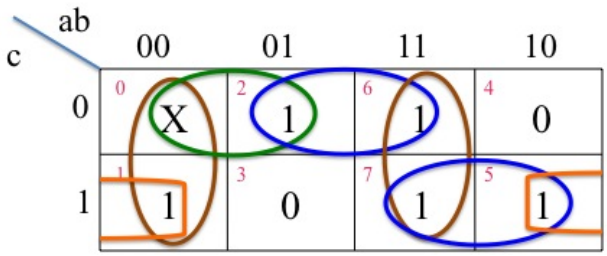
$$\begin{aligned}
\text{LHS: f(a,b,c,d)} = \; & (a' + c)(a' + b' + d)(b' + c' + d) \\
& = (c + f(a, b, 0, d)) \cdot (c' + f(a, b, 1, d)) \\
& = (c + a'(a' + b' + d)) \cdot (c' + (a' + b' + d) \cdot (b' + d)) \\
& = (c + a'(1 + b' + d)) \cdot (c' + (a'.0 + b' + d)) \\
& = (c + a') \cdot (c' + b' + d)
\end{aligned}$$

Rubric:
Deduct 2 points for each incorrect steps (max deduction 15 pts).
Deduct 5 points if any method other than Shannon's is used.

5 Karnaugh Map to Reduced Sum of Product

I. Use Karnaugh map to simplify the following switching function. List all possible minimal *sum of product* expressions. There is no need to draw the circuit diagram.

$f(a,b,c) = \sum m(1,2,5,6,7) + \sum d(0)$ **(25pts)**



Prime Implicants:

1. $\sum m(0,2) = a'c'$
2. $\sum m(0,1) = a'b'$
3. $\sum m(2,6) = bc'$
4. $\sum m(6,7) = ab$
5. $\sum m(1,5) = b'c$
6. $\sum m(5,7) = ac$

There are no Essential Prime Implicants.

Minimal Sum of Product Expresssions:

1. $f(a,b,c) = \sum m(2,6) + \sum m(6,7) + \sum m(1,5)$
2. $f(a,b,c) = \sum m(0,2) + \sum m(6,7) + \sum m(1,5)$
3. $f(a,b,c) = \sum m(2,6) + \sum m(5,7) + \sum m(1,5)$
4. $f(a,b,c) = \sum m(2,6) + \sum m(5,7) + \sum m(0,1)$

Rubric:

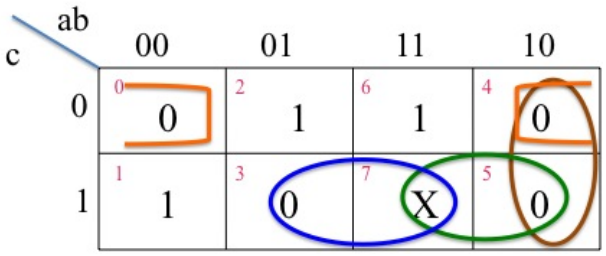
If all expression are correct full points (25) For each wrong expression deduct 6 points. at this point if total is below 5 pts and the K-map is shown and is correct then give 8 points else give 5 points (total) for attempting.

Okay if expression is shown in terms of implicants and not literals

6 Karnaugh Map to Reduced Product of Sum

Use Karnaugh map to simplify the following switching function. List all possible minimal *product of sum* expressions. There is no need to draw the circuit diagram. **(25pts)**

$f(a,b,c) = \sum m(1,2,6) + \sum d(7)$



Prime Implicates:

1. $\prod M(0,4) = b + c$
2. $\prod M(3,7) = b' + c'$
3. $\prod M(4,5) = a' + b$
4. $\prod M(5,7) = a' + c'$

Essential Prime Implicates:

1. $\prod M(3,7)$
2. $\prod M(0,4)$

Minimal Product of Sum Expresssions:

1. $f(a,b,c) = \prod M(0,4).\prod M(3,7).\prod M(4,5)$
2. $f(a,b,c) = \prod M(0,4).\prod M(3,7).\prod M(5,7)$

Rubric:
If all expression are correct full points (25) For each wrong expression deduct 12 points. at this point if total is below 12 pts and the K-map is shown and is correct then give 8 points for the K-map else give 5 points(total) for attempting.
Okay if expression is shown in terms of implicates and not literals