Midterm 1 Solution

I

Truth table of $f(a, b, c, d)$.

id	a	b	c	d	f
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

Canonical product-of-sums format for f.

$$f \quad = \quad (a+b+c+d)(a+b+c+d')(a+b+c'+d)(a+b'+c+d)(a'+b+c+d)$$

Rubric:

- Deduct 0 point for further simplifying the canonical form.
- Deduct 1 point for each missing or incorrect entry of the truth table.
- Deduct 1 point for each missing or incorrect maxterm of the canonical form.
- Deduct 2 points for only giving the simplified boolean expression.
- Deduct 3 points for incorrect simplified boolean expression.
- Deduct 5 points for only giving $\prod M(0, 1, 2, 4, 8)$.

II

Prove Consensus Theorem for $a'b + a'c'd + b'c'd = a'b + b'c'd$.

Proof:

$$\begin{aligned}
a'b + b'c'd + a'c'd &\overset{identity}{=} a'b + b'c'd + a'c'd \cdot (\mathbf{1}) \\
&\overset{complements}{=} a'b + b'c'd + a'c'd \cdot (\mathbf{b} + \mathbf{b'}) \\
&\overset{distributivity}{=} a'b + b'c'd + a'\mathbf{b}c'd + a'\mathbf{b'}c'd \\
&= \underbrace{a'b + a'bc'd}_{covering} + \underbrace{b'c'd + a'b'c'd}_{covering} \\
&= a'b + b'c'd
\end{aligned}$$

Rubric:

- Deduct 1 point for not listing rules.
- Deduct 8 points for incorrect derivation.
- Deduct 10 pionts for using only Consesus Theorem to prove.

III Prove Shannon's expansion, $bf(a,1,c) + b'f(a,0,c) = (b + f(a,0,c))(b' + f(a,1,c))$

Proof:

$$\begin{aligned}(b + f(a,0,c))(b' + f(a,1,c)) &\overset{distributivity}{=} bb' + bf(a,1,c) + b'f(a,0,c) + f(a,1,c)f(a,0,c)\\ &\overset{complements}{=} \mathbf{0} + bf(a,1,c) + b'f(a,0,c) + f(a,1,c)f(a,0,c)\\ &\overset{consensus}{=} bf(a,1,c) + b'f(a,0,c)\end{aligned}$$

Rubric:

- Deduct 1 point for not listing rules.
- Deduct 3 points for assuming $f(a,0,c)' = f(a,1,c)$.
- Deduct 5 points for using switching function.
- Deduct 8 points for not using Boolean algebra and switching function.

IV Possible minimal sum of products:

$\Sigma m(0,2) + \Sigma m(3,7) + \Sigma m(4,5)$ →a'c'+bc+ab'

$\Sigma m(0,2) + \Sigma m(3,7) + \Sigma m(0,4)$ →a'c'+bc+b'c'

$\Sigma m(2,3) + \Sigma m(3,7) + \Sigma m(4,5)$ →a'b+bc+ab'

$\Sigma m(2,3) + \Sigma m(3,7) + \Sigma m(0,4)$ →a'b+bc+b'c'

$\Sigma m(2,3) + \Sigma m(5,7) + \Sigma m(4,5)$ →a'b+ac+ab'

$\Sigma m(2,3) + \Sigma m(5,7) + \Sigma m(0,4)$ →a'b+ac+b'c'

Rubric:

- Award 7 points if correct K-map is drawn
- Deduct 3 points for every solution that is missing
- Deduct 1 point for every term in a given solution found incoorect or missing

V Possible minimal sum of products:

$\Pi M(0,2)\Pi M(3,7)\Pi M(4,5)$ →(a+c)(b'+c')(a'+b)

$\Pi M(0,2)\Pi M(3,7)\Pi M(0,4)$ →(a+c)(b'+c')(b+c)

$\Pi M(2,3)\Pi M(3,7)\Pi M(4,5)$ →(a+b')(b'+c')(a'+b)

$\Pi M(2,3)\Pi M(3,7)\Pi M(0,4)$ →(a+b')(b'+c')(b+c)

$\Pi M(2,3)\Pi M(5,7)\Pi m(4,5)$ →(a+b')(a'+c')(a'+b)

$\Pi M(2,3)\Pi M(5,7)\Pi M(0,4)$ →(a+b')(a'+c')(b+c)

Rubric:

- Award 7 points if correct K-map is drawn
- Deduct 3 points for every solution that is missing
- Deduct 1 point for every term in a given solution found incoorect or missing