CSE 140
Discussion Session #1

Topics

- ✓Consensus Theorem
- ✓Shannon’s Expansion
- ✓Truth Table, Canonical Forms and Combinational Circuits

Consensus Theorem

- In Sum of Products (SOP) Form
  - $AB + B'C + AC = AB + B'C$

- In Product of Sums (POS) Form
  - $(A + B)(B' + C)(A + C) = (A + B)(B' + C)$

Proof of Consensus Theorem (POS)

- $(A + B)(B' + C)(A + C) = (A + B)(B' + C)$

L.H.S. $= (A + B)(B' + C)(A + C)$

$= (A + B)(B' + C)(A + C + 0)$ // (Identity)

$= (A + B)(B' + C)(A + BB' + C)$ // Complements

$= (A + B)(B' + C)\ (A + B + C)(A + B' + C)$ // Distributive

$= (A + B)\ (A + B + C)\ (B' + C)\ (A + B' + C)$ // Commutative

$= (A + B)\ (1 + C)\ (B' + C)\ (A + 1)$ // Distributive

$= (A + B)\ (1)\ (B' + C)\ (1)$ // Identity

$= (A + B)(B' + C)$

$(A + B)(B' + C)(A + C) = (A + B)(B' + C)$

Consensus Theorem Visualization using Truth Table (POS)

- (A + B)(B’ + C)(A + C) = (A + B)(B’ + C)
- F(A,B,C) = (A + B)(B’ + C)(A + C)
- G(A,B,C) = (A + B)(B’ + C)

A	B	C	A + B	B’+ C	A + C	F	G
0	0	0	0	1	0	0	0
0	0	1	0	1	1	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1
1	0	1	1	1	1	1	1
1	1	0	1	0	1	0	0
1	1	1	1	1	1	1	1

Consensus Theorem Examples

- In Sum of Products Form
  - AB + B’CD’ = AB + B’CD’ + ACD’
  - AC’D + BCE = AC’D + BCE + ABDE
- In Product of Sums Form
  - (A + B)(B’ + C + D’) = (A + B)(B’ + C + D’)(A + C + D’)
  - (A + B + C’)(C + D + E’) = (A + B + C’)(C + D + E’)(A + B + D + E’)

Shannon’s Expansion

- $f(A,B,C) = A.f(1,B,C) + A'.f(0,B,C) = B.f(A,1,C) + B'.f(A,0,C)$

$$
\begin{aligned}
f(A,B,C) &= A.f(1,B,C) + A'.f(0,B,C) \\
&= A.(\,B.f(1,1,C) + B'.f(1,0,C)) + A'.(B.f(0,1,C) + B'.f(0,0,C)\,) \\
&= AB.f(1,1,C) + AB'.f(1,0,C) + A'B.f(0,1,C) + A'B'.f(0,0,C)
\end{aligned}
$$

- $f(A,B,C) = (\,A + f(0,B,C)\,).(\,A' + f(1,B,C)\,) = (\,B + f(A,0,C)\,).(\,B' + f(A,1,C)\,)$

$$
\begin{aligned}
f(A,B,C) &= (\,A + f(0,B,C)\,).(\,A' + f(1,B,C)\,) \\
&= (\,A + (B + f(0,0,C)).(\,B' + f(0,1,C))).(\,A' + (B + f(1,0,C)).(\,B' + f(1,1,C))) \\
&= (\,A + B + f(0,0,C)).(\,A + B' + f(0,1,C)).(\,A' + B + f(1,0,C)).(\,A'+ B' + f(1,1,C))
\end{aligned}
$$

Consensus Using Shannon's Expansion (SOP)

- $AB + B'C + AC = AB + B'C$

$$f(A,B,C) = AB + B'C + AC$$

$$f(A,B,C) = B.f(A,1,C) + B'.f(A,0,C)$$

$$f(A,1,C) = A.1 + 0.C + AC = A + AC = A(1+C) = A$$

$$f(A,0,C) = A.0 + 1.C + AC = C + AC = C(1+A) = C$$

$$f(A,B,C) = B.f(A,1,C) + B'.f(A,0,C)$$

$$= B.A + B'C$$

$$= AB + B'C$$

$$\mathbf{f(A,B,C) = AB + B'C + AC = AB + B'C}$$

Consensus Using Shannon’s Expansion (POS)

- $(A + B)(B' + C)(A + C) = (A + B)(B' + C)$

$$f(A,B,C) = (A + B)(B' + C)(A + C)$$

$$f(A,B,C) = (B + f(A,0,C)).(B' + f(A,1,C))$$

$$f(A,0,C) = (A + 0)(1 + C)(A + C) = (A).(1)(A+C) = A + AC = A(1+C) = A$$

$$f(A,1,C) = (A + 1)(0 + C)(A + C) = (1).(C)(A+C) = AC + C = C(A+1) = C$$

$$\begin{aligned} f(A,B,C) &= (B + f(A,0,C)).(B' + f(A,1,C)) \\ &= (B + A).(B' + C) \\ &= (A + B)(B' + C) \end{aligned}$$

$$\mathbf{f(A,B,C) = (A + B)(B' + C)(A + C) = (A + B)(B' + C)}$$

Truth Tables and Circuits

- A machine inputs 3 binary bits (X2,X1,X0) and outputs Y =1 when the number of bits are less than or equal to 1. Otherwise, the output is Y=0.

Truth Tables, Canonical Forms and Circuits

X2	X1	X0	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

- Sum of Products

  $f(X2,X1,X0) = X2'.X1'.X0' + X2'.X1'.X0 + X2'.X1.X0' + X2.X1'.X0'$

  $= \Sigma\, m(0,1,2,4)$

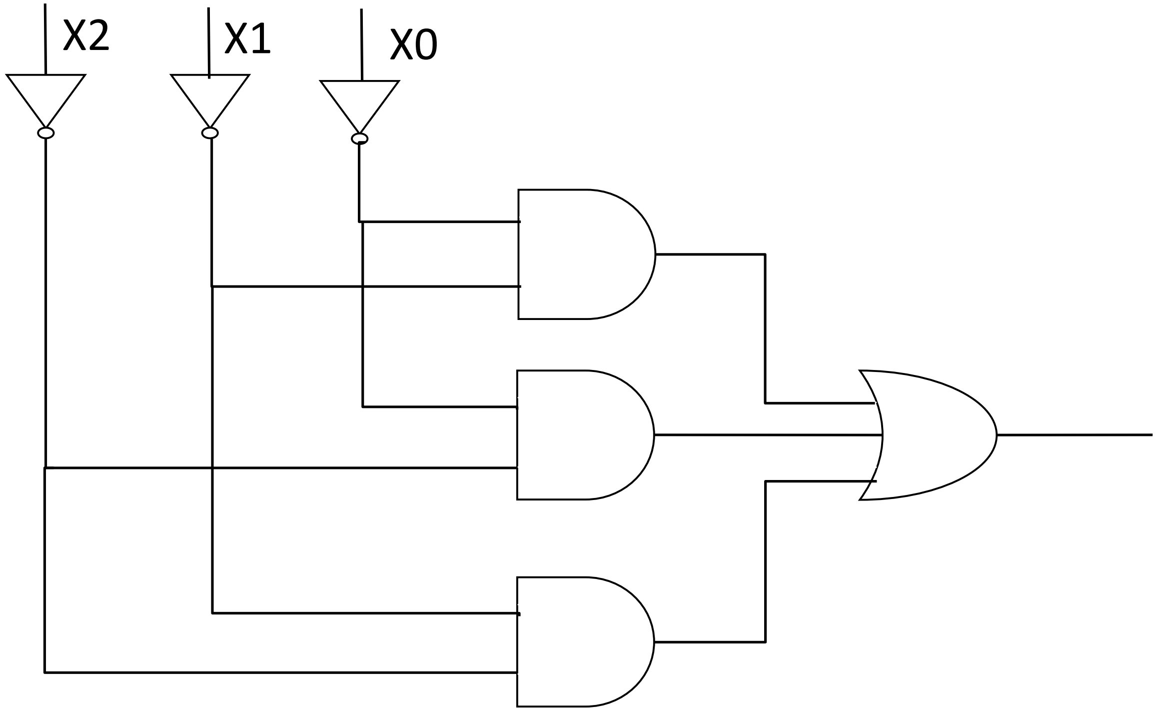
- Product of Sums

  $f(X2,X1,X0) = (X2 + X1' + X0')(X2' + X1 + X0')(X2' + X1' + X0)(X2' + X1' + X0')$

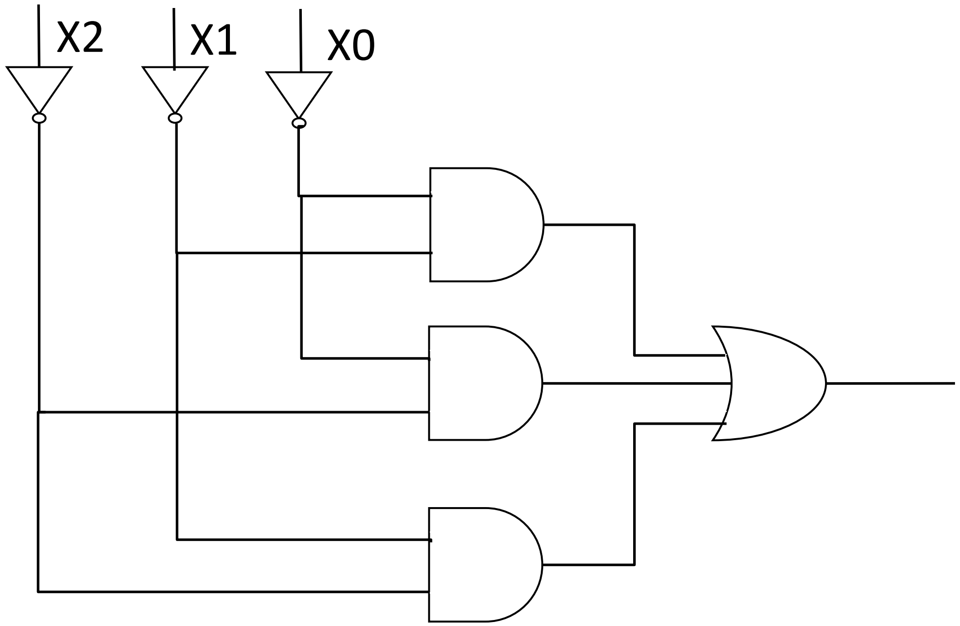
  $= \Pi\, M(3,5,6,7)$

$$\begin{aligned} f(X2,X1,X0) &= X2'.X1'.X0' + X2'.X1'.X0 + X2'.X1.X0' + X2.X1'.X0' \\ &= X2'.X1'.X0 + X2'.X1'.X0' + X2'.X1.X0' + X2'.X1'.X0' + X2.X1'.X0' + X2'.X1'.X0' \\ &= X2'.X1'(X0 + X0') + X2'.X0'(X1 + X1') + X1'.X0'(X2 + X2') \\ &= X0'X1' + X1' X2' + X0'X2' \end{aligned}$$

Truth Tables, Canonical Forms and Circuits



Truth Tables, Canonical Forms and Circuits



Variables	3 (X0,X1,X2)
Literals	3 (X0',X1',X2')
Operators	11(3 AND, 2 OR, 6 NOT)
Gates	7 (3 AND, 1 OR, 3 NOT)
Nets	10
Pins	19

f(X2,X1,X0) = X0'X1' + X0'X2' + X1'X2'

Thank You !

- Remember:
  - Homework 1 due on Tuesday 1/12 11:59 PM