HW3 preparation

- Universal Gates
- Other gates, properties and usage
- Simplifying using shannon expansion
- SR Latch

Universal gates

We learnt how to implement all switching functions using {AND, OR, NOT} gates. Now we want to check if there is other sets using which we can implement all switching functions.

Universal Set: A set of gates such that any switching function can be implemented with the gates in the set.

Approach: check to see if we can implement each of the following gates using gates in the set:

- **AND**
- **OR**
- **NOT**

Examples

- {AND, OR, NOT}, Universal
- {AND, NOT}, Universal
  - NOT
  - AND
  - OR : (X’ AND Y’)’ = X OR Y
- {OR, NOT}, Universal
  - NOT
  - OR
  - AND : (X’ OR Y’)’ = X AND Y

Examples

- {AND, OR}, not universal
  - AND
  - OR
  - NOT : we cannot built NOT gate using only {AND, OR}
- {XOR}, not universal
  - NOT : X XOR 1 = X' = NOT X
  - OR : we cannot built OR gate using only {XOR}
  - AND : we cannot built AND gate using only {XOR}
- {XNOR}, not universal
  - NOT : X XNOR 0 = X' = NOT X
  - OR : we cannot built OR gate using only {XNOR}
  - AND : we cannot built AND gate using only {XNOR}

$x \text{ xor } y = x'y + xy'$

$x \text{ xnor } y = xy + x'y'$

Examples

- {XOR, OR}, Universal
  - NOT : X XOR 1 = X’ = NOT X
  - OR
  - AND : (X’ OR Y’)’ = XY
- {XOR, AND}, Universal
  - NOT : X XOR 1 = X’ = NOT X
  - AND
  - OR : (X’ AND Y’)’ = X AND Y

use the XOR gate to build NOT, and then using NOT and OR we can build AND .

we saw how we can build AND from {OR,NOT}

use the XOR gate to build NOT, and then using NOT and AND we can build OR .

Examples

- {XNOR, OR}, Universal
  - NOT : X XNOR 0 = X’ = NOT X
  - OR
  - AND : (X’ OR Y’)’ = X AND Y
- {XNOR, AND}, Universal
  - NOT : X XNOR 0 = X’ = NOT X
  - AND
  - OR : (X’ AND y’)’ = X OR Y

use the XNOR gate to build NOT, and then using NOT and OR we can build AND .

use the XNOR gate to build NOT, and then using NOT and AND we can build AND .

Examples

- {NAND}, Universal
  - NOT : X NAND 1 = X’ = NOT X
  - AND : (X NAND Y)’ = X AND Y
  - OR : (X’ NAND Y’) = X OR Y
- {NOR}, Universal
  - NOT : X NOR 0 = X’ = NOT X
  - AND : (X’ NOR Y’) = X AND Y
  - OR : (X NOR Y)’ = X OR Y

If a set is universal, after adding extra gates to the set, it will remain universal!

- {NOR, XOR}, universal
- {NAND, AND }, universal

Examples

fixing the first input to “0”, the output will be AND of the last two inputs.

f(a, b, c) = (a + b)(a + c), not universal

- AND : f(0, B, C) = (0 + B)(0 + C) = BC = B AND C
- OR : f(A, B, 1) = (A + B)(A + 1) = A + B = A OR B
- NOT gate : NOT gate cannot be built with the given function.
  - you can see no complement has been used in the function, so there is no way to build a NOT gate from the given function

setting the third input to “1”, the output will be OR of the first two inputs.

Examples

$f(a, b, c) = ab + ac + a'c'$ universal

- NOT : $f(0, 0, C) = 0 \cdot 0 + 0 \cdot C + 1 \cdot C' = C'$ = NOT C
- OR : $f(1,B, C) = 1 \cdot B + 1 \cdot C + 0 \cdot C' = B + C$ = B OR C
- AND : AND gate can be built with NOT and OR gates.

setting the first input to “1”, the output will be OR of the second and third inputs

fixing the first and second input to “0”, the output will be NOT of the third input.

XOR

- commutative:

  $x \oplus y = y \oplus x$

- Associative:

  $(x \oplus y) \oplus z = x \oplus (y \oplus z)$

- $x \oplus 1 = x'$,, $x \oplus 0 = x$
- $x \oplus x = 0$,, $x \oplus x' = 1$

xor of two values are 1 *iff* they are not equal

xor of bunch of zeros is zero!

$0 \oplus 0 \oplus 0 = 0$

$0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 \oplus 0 = 0$

xor of odd numbers of ones is, one.

$0 \oplus 1 = 1$

$0 \oplus 0 \oplus 1 = 1$

$1 \oplus 1 \oplus 1 = 1$

xor of even numbers of ones is, zero.

$0 \oplus 1 \oplus 1 = 0$

$1 \oplus 0 \oplus 1 \oplus 0 \oplus 1 \oplus 0 = 1$

$1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 = 1$

Distributivity?!

$$a \oplus (b + c) \stackrel{?}{=} (a \oplus b) + (a \oplus c)$$

$LHS = a \oplus (b + c) = a(b + c)' + a'(b + c) = ab'c' + a'b + a'c$

$RHS = (a \oplus b) + (a \oplus c) = ab' + a'b + ac' + a'c = a'b + a'c + a(b' + c')$

LHS and RHS are not equal. Hence, XOR is not distributive over OR.

As a counterexample, if $(a, b, c) = (1, 0, 1)$, $LHS=0$ and $RHS=1$.

Distributivity?!

$a + (b \oplus c) \overset{?}{=} (a + b) \oplus (a + c)$

LHS = $a + (b \oplus c) = a + (bc' + b'c) =$

RHS = $(a + b) \oplus (a + c) = (a + b)(a + c)' + (a + b)'(a + c) = a'c'(a + b) + a'b'(a + c) = a'c'a + a'c'b + a'b'a + a'b'c = a'c'b + a'b'c = a'(bc' + b'c)$

LHS and RHS are not equal. Hence, OR is not distributive over XOR.

As a counterexample, if $(a, b, c) = (1, 0, 0)$, LHS=1 and RHS=0.

simplify the given function:

$f(a,b,c) = x \oplus (y' + z) \oplus xy' \oplus yx$

Shannon Theorem : $f(x,y,z) = x.f(1,y,z) + x'.f(0,y,z)$

$= y.f(x,1,z) + y'.f(x,0,z)$

$= z.f(x,y,1) + z'.f(x,y,0)$

simplify the given function:

$f(a,b,c) = x \oplus (y' + z) \oplus xy' \oplus yx$

Shannon Theorem : $f(x,y,z) = y.f(x,1,z) + y'.f(x,0,z)$

$f(x,1,z) = x \oplus (0 + z) \oplus x0 \oplus 1x$

$= x \oplus z \oplus 0 \oplus x$ //associativity, comutativity

$= (x \oplus x) \oplus z \oplus 0$

$= 0 \oplus z \oplus 0$

$= z$

simplify the given function:

$f(a,b,c) = x \oplus (y' + z) \oplus xy' \oplus yx$

Shannon Theorem : $f(x,y,z) = y.f(x,1,z) + y'.f(x,0,z)$

$f (x,0,z) = x \oplus (1 + z) \oplus x1 \oplus 0x$

$= x \oplus 1 \oplus x \oplus 0$

$= (x \oplus x) \oplus (1 \oplus 0)$

$= (0) \oplus (1)$

$= 1$

simplify the given function:

$f(a,b,c) = x \oplus (y' + z) \oplus xy' \oplus yx$

Shannon Theorem : $f(x,y,z) = y.f(x,1,z) + y'.f(x,0,z)$

$f(x,1,z) = z$

$f(x,0,z) = 1$

$f(x,y,z) = yz + y'$

$= yz + y' + z$ //consensus

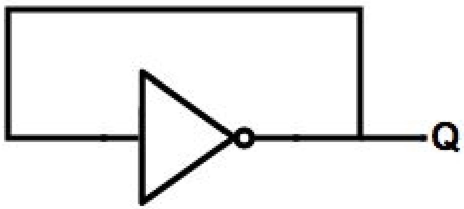
$= z(y+1) + y'$

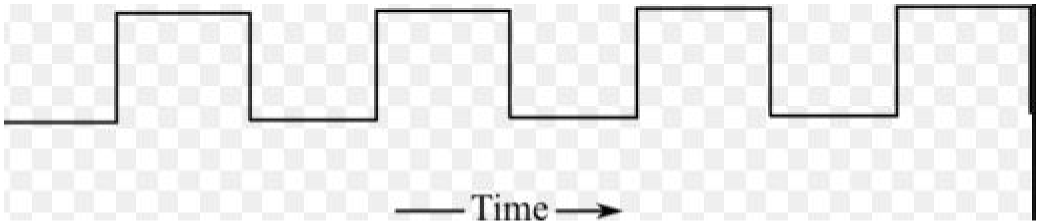
$= z + y'$

Sequential Circuits

The output depends on the *current input* and the *previous output*.

Feedback loops:

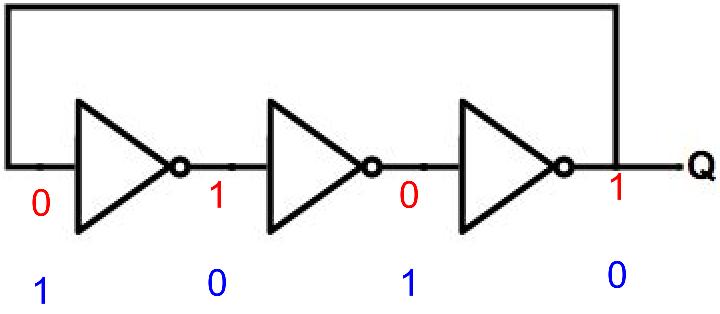


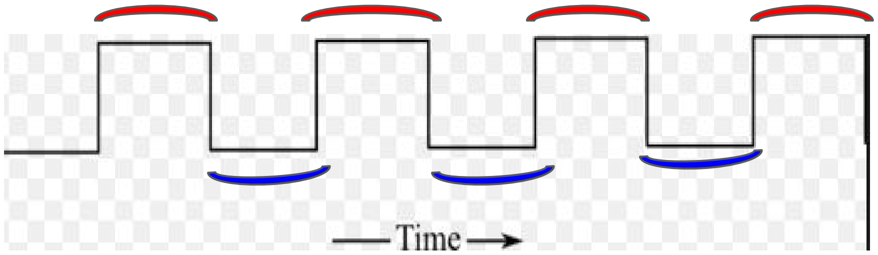


The output keeps oscillating.

what will happen if we add more inverters?

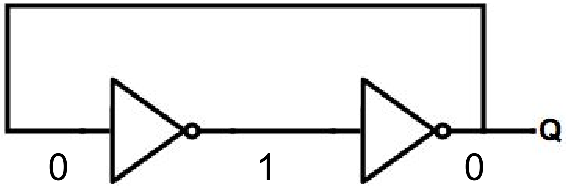
Feedback loops



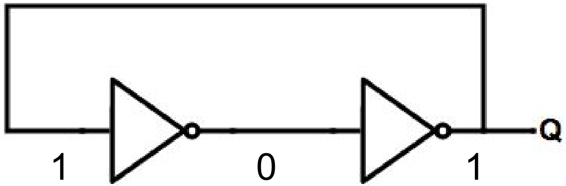


The situation will be same if we have *"odd"* number of inverters

what if we have “even” number of inverters ?



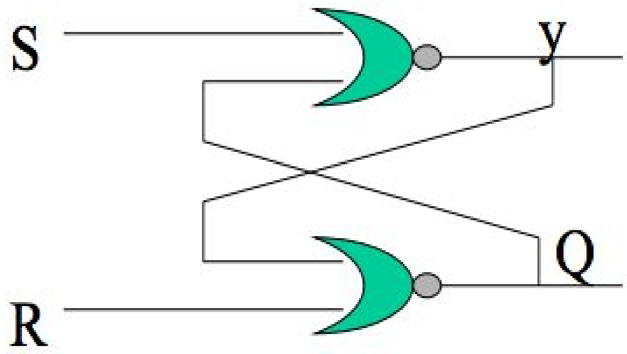
Starting from 0, it will keep value 0.



Starting from 1, it will keep value 1.

It behaves like memory, but there is no input to control it.

SR Latch Analysis



$$y = (S+Q)'$$

$$Q = (R+y)'$$

Id	Q(t)	y(t)	S	R	$Q(t_1)$	$y(t_1)$	$Q(t_2)$	$y(t_2)$	$Q(t_3)$	$y(t_3)$
0	0	0	0	0	1	1	0	0	1	1
1	0	0	0	1	0	1	0	1	0	1
2	0	0	1	0	1	0	1	0	1	0
3	0	0	1	1	0	0	0	0	0	0
4	0	1	0	0	0	1	0	1	0	1
5	0	1	0	1	0	1	0	1	0	1
6	0	1	1	0	0	0	1	0	1	0
7	0	1	1	1	0	0	0	0	0	0
8	1	0	0	0	1	0	1	0	1	0
9	1	0	0	1	0	0	0	1	0	1
10	1	0	1	0	1	0	1	0	1	0
11	1	0	1	1	0	0	0	0	0	0
12	1	1	0	0	0	0	1	1	0	0
13	1	1	0	1	0	0	0	1	0	1
14	1	1	1	0	0	0	1	0	1	0
15	1	1	1	1	0	0	0	0	0	0

$y(t+1) = (S+Q(t))'$

$Q(t+1) = (R+ y(t))'$

$S = 0, R = 0 \rightarrow y(t+1) = Q'(t) , Q(t+1) = y'(t)$

$S = 0, R = 1 \rightarrow y(t+1) = Q'(t) , Q(t+1) = 0$

$S = 1, R = 0 \rightarrow y(t+1) = 0 , Q(t+1) = y'(t)$

$S = 1, R = 1 \rightarrow y(t+1) = 0 , Q(t+1) = 0$

Id	Q(t)	y(t)	S	R	$Q(t_1)$	$y(t_1)$	$Q(t_2)$	$y(t_2)$	$Q(t_3)$	$y(t_3)$
0	0	0	0	0	1	1	0	0	1	1
1	0	0	0	1	0	1	0	1	0	1
2	0	0	1	0	1	0	1	0	1	0
3	0	0	1	1	0	0	0	0	0	0
4	0	1	0	0	0	1	0	1	0	1
5	0	1	0	1	0	1	0	1	0	1
6	0	1	1	0	0	0	1	0	1	0
7	0	1	1	1	0	0	0	0	0	0
8	1	0	0	0	1	0	1	0	1	0
9	1	0	0	1	0	0	0	1	0	1
10	1	0	1	0	1	0	1	0	1	0
11	1	0	1	1	0	0	0	0	0	0
12	1	1	0	0	0	0	1	1	0	0
13	1	1	0	1	0	0	0	1	0	1
14	1	1	1	0	0	0	1	0	1	0
15	1	1	1	1	0	0	0	0	0	0

$$y(t+1) = (S+Q(t))'$$

$$Q(t+1) = (R+ y(t))'$$

S = 0, R = 0 → y(t+1) = Q'(t) , Q(t+1) = y'(t)

S = 0, R = 1 → y(t+1) = Q'(t) , Q(t+1) =0

S = 1, R = 0 → y(t+1) = 0 , Q(t+1) = y'(t)

S = 1, R = 1 → y(t+1) = 0 , Q(t+1) = 0

Id	Q(t)	y(t)	S	R	$Q(t_1)$	$y(t_1)$	$Q(t_2)$	$y(t_2)$	$Q(t_3)$	$y(t_3)$
0	0	0	0	0	1	1	0	0	1	1
1	0	0	0	1	0	1	0	1	0	1
2	0	0	1	0	1	0	1	0	1	0
3	0	0	1	1	0	0	0	0	0	0
4	0	1	0	0	0	1	0	1	0	1
5	0	1	0	1	0	1	0	1	0	1
6	0	1	1	0	0	0	1	0	1	0
7	0	1	1	1	0	0	0	0	0	0
8	1	0	0	0	1	0	1	0	1	0
9	1	0	0	1	0	0	0	1	0	1
10	1	0	1	0	1	0	1	0	1	0
11	1	0	1	1	0	0	0	0	0	0
12	1	1	0	0	0	0	1	1	0	0
13	1	1	0	1	0	0	0	1	0	1
14	1	1	1	0	0	0	1	0	1	0
15	1	1	1	1	0	0	0	0	0	0

$y(t+1) = (S+Q(t))'$

$Q(t+1) = (R+ y(t))'$

$S = 0, R = 0 \rightarrow y(t+1) = Q'(t)\ , Q(t+1) = y'(t)$

$S = 0, R = 1 \rightarrow y(t+1) = Q'(t)\ , Q(t+1) =0$

$S = 1, R = 0 \rightarrow y(t+1) = 0\ , Q(t+1) = y'(t)$

$S = 1, R = 1 \rightarrow y(t+1) = 0\ , Q(t+1) = 0$

Id	Q(t)	y(t)	S	R	$Q(t_1)$	$y(t_1)$	$Q(t_2)$	$y(t_2)$	$Q(t_3)$	$y(t_3)$
0	0	0	0	0	1	1	0	0	1	1
1	0	0	0	1	0	1	0	1	0	1
2	0	0	1	0	1	0	1	0	1	0
3	0	0	1	1	0	0	0	0	0	0
4	0	1	0	0	0	1	0	1	0	1
5	0	1	0	1	0	1	0	1	0	1
6	0	1	1	0	0	0	1	0	1	0
7	0	1	1	1	0	0	0	0	0	0
8	1	0	0	0	1	0	1	0	1	0
9	1	0	0	1	0	0	0	1	0	1
10	1	0	1	0	1	0	1	0	1	0
11	1	0	1	1	0	0	0	0	0	0
12	1	1	0	0	0	0	1	1	0	0
13	1	1	0	1	0	0	0	1	0	1
14	1	1	1	0	0	0	1	0	1	0
15	1	1	1	1	0	0	0	0	0	0

y(t+1) = (S+Q(t))’

Q(t+1) = (R+ y(t))’

S = 0, R = 0 → y(t+1) = Q’(t) , Q(t+1) = y’(t)

S = 0, R = 1 → y(t+1) = Q’(t) , Q(t+1) =0

S = 1, R = 0 → y(t+1) = 0 , Q(t+1) = y’(t)

S = 1, R = 1 → y(t+1) = 0 , Q(t+1) = 0

Id	$Q(t)$	$y(t)$	S	R	$Q(t_1)$	$y(t_1)$	$Q(t_2)$	$y(t_2)$	$Q(t_3)$	$y(t_3)$
0	0	0	0	0	1	1	0	0	1	1
1	0	0	0	1	0	1	0	1	0	1
2	0	0	1	0	1	0	1	0	1	0
3	0	0	1	1	0	0	0	0	0	0
4	0	1	0	0	0	1	0	1	0	1
5	0	1	0	1	0	1	0	1	0	1
6	0	1	1	0	0	0	1	0	1	0
7	0	1	1	1	0	0	0	0	0	0
8	1	0	0	0	1	0	1	0	1	0
9	1	0	0	1	0	0	0	1	0	1
10	1	0	1	0	1	0	1	0	1	0
11	1	0	1	1	0	0	0	0	0	0
12	1	1	0	0	0	0	1	1	0	0
13	1	1	0	1	0	0	0	1	0	1
14	1	1	1	0	0	0	1	0	1	0
15	1	1	1	1	0	0	0	0	0	0

$y(t+1) = (S+Q(t))'$

$Q(t+1) = (R+ y(t))'$

S = 0, R = 0 → $y(t+1) = Q'(t)$, $Q(t+1) = y'(t)$

S = 0, R = 1 → $y(t+1) = Q'(t)$, $Q(t+1) = 0$

S = 1, R = 0 → $y(t+1) = 0$, $Q(t+1) = y'(t)$

S = 1, R = 1 → $y(t+1) = 0$, $Q(t+1) = 0$

Id	$Q(t)$	$y(t)$	S	R	$Q(t_1)$	$y(t_1)$	$Q(t_2)$	$y(t_2)$	$Q(t_3)$	$y(t_3)$
0	0	0	0	0	1	1	0	0	1	1
1	0	0	0	1	0	1	0	1	0	1
2	0	0	1	0	1	0	1	0	1	0
3	0	0	1	1	0	0	0	0	0	0
4	0	1	0	0	0	1	0	1	0	1
5	0	1	0	1	0	1	0	1	0	1
6	0	1	1	0	0	0	1	0	1	0
7	0	1	1	1	0	0	0	0	0	0
8	1	0	0	0	1	0	1	0	1	0
9	1	0	0	1	0	0	0	1	0	1
10	1	0	1	0	1	0	1	0	1	0
11	1	0	1	1	0	0	0	0	0	0
12	1	1	0	0	0	0	1	1	0	0
13	1	1	0	1	0	0	0	1	0	1
14	1	1	1	0	0	0	1	0	1	0
15	1	1	1	1	0	0	0	0	0	0

$y(t+1) = (S+Q(t))'$

$Q(t+1) = (R+ y(t))'$

$S = 0, R = 0 \rightarrow y(t+1) = Q'(t)\ , Q(t+1) = y'(t)$

$S = 0, R = 1 \rightarrow y(t+1) = Q'(t)\ , Q(t+1) = 0$

$S = 1, R = 0 \rightarrow y(t+1) = 0\ , Q(t+1) = y'(t)$

$S = 1, R = 1 \rightarrow y(t+1) = 0\ , Q(t+1) = 0$

Id	Q(t)	y(t)	S	R	$Q(t_1)$	$y(t_1)$	$Q(t_2)$	$y(t_2)$	$Q(t_3)$	$y(t_3)$
0	0	0	0	0	1	1	0	0	1	1
1	0	0	0	1	0	1	0	1	0	1
2	0	0	1	0	1	0	1	0	1	0
3	0	0	1	1	0	0	0	0	0	0
4	0	1	0	0	0	1	0	1	0	1
5	0	1	0	1	0	1	0	1	0	1
6	0	1	1	0	0	0	1	0	1	0
7	0	1	1	1	0	0	0	0	0	0
8	1	0	0	0	1	0	1	0	1	0
9	1	0	0	1	0	0	0	1	0	1
10	1	0	1	0	1	0	1	0	1	0
11	1	0	1	1	0	0	0	0	0	0
12	1	1	0	0	0	0	1	1	0	0
13	1	1	0	1	0	0	0	1	0	1
14	1	1	1	0	0	0	1	0	1	0
15	1	1	1	1	0	0	0	0	0	0

$y(t+1) = (S+Q(t))'$

$Q(t+1) = (R+ y(t))'$

$S = 0, R = 0 \rightarrow y(t+1) = Q'(t)\ ,\ Q(t+1) = y'(t)$

$S = 0, R = 1 \rightarrow y(t+1) = Q'(t)\ ,\ Q(t+1) = 0$

$S = 1, R = 0 \rightarrow y(t+1) = 0\ ,\ Q(t+1) = y'(t)$

$S = 1, R = 1 \rightarrow y(t+1) = 0\ ,\ Q(t+1) = 0$

Id	Q(t)	y(t)	S	R	$Q(t_1)$	$y(t_1)$	$Q(t_2)$	$y(t_2)$	$Q(t_3)$	$y(t_3)$
0	0	0	0	0	1	1	0	0	1	1
1	0	0	0	1	0	1	0	1	0	1
2	0	0	1	0	1	0	1	0	1	0
3	0	0	1	1	0	0	0	0	0	0
4	0	1	0	0	0	1	0	1	0	1
5	0	1	0	1	0	1	0	1	0	1
6	0	1	1	0	0	0	1	0	1	0
7	0	1	1	1	0	0	0	0	0	0
8	1	0	0	0	1	0	1	0	1	0
9	1	0	0	1	0	0	0	1	0	1
10	1	0	1	0	1	0	1	0	1	0
11	1	0	1	1	0	0	0	0	0	0
12	1	1	0	0	0	0	1	1	0	0
13	1	1	0	1	0	0	0	1	0	1
14	1	1	1	0	0	0	1	0	1	0
15	1	1	1	1	0	0	0	0	0	0

circles: state
arrows: SR input
Q(t)y(t) = current state
Q(t3)y(t3) = next state (state)