CSE 255 – Lecture 7

Data Mining and Predictive Analytics

Rich-club phenomena

Rich clubs

Rich nodes, and rich-clubs

- We’ve seen how real-world networks are characterized by “rich-get-richer” phenomena, and that networks tend to follow power-law distributions, i.e., a small number of nodes have high degree, while many nodes have low degree

- But what about the connectivity structure **between** high-degree nodes?
  - i.e., are nodes of high degree likely to connect to other nodes of high degree?

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Rich nodes, and rich-clubs

Why is this characterization important?

- Is a power-grid robust to cascading failure? Maybe we would want "hubs" in such a network **not** to be connected to other hubs
- Do people who write many papers tend to co-author with others who write many papers, or do they form distinct authorship "hierarchies"?
- High-degree nodes in Protein-Protein-Interaction networks may perform regulatory functions. Whether high-degree nodes are interconnected may determine whether they regulate different functional modules

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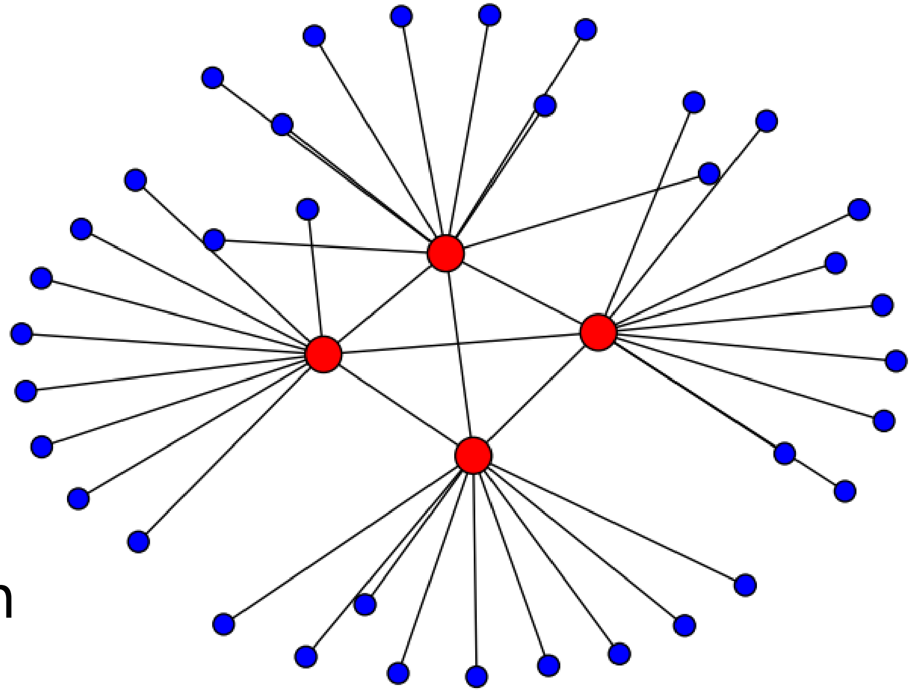
e.g. (from Wikipedia)



red nodes form a "rich club"

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Let’s try to characterize this property

What fraction of edges appear between high-degree nodes?

#edges between nodes of degree > k

$$\phi(k) = \frac{2E_{>k}}{N_{>k}(N_{>k}-1)}$$

degree

#nodes of degree > k

Whether this function **increases** or **decreases** with *k* will somehow characterize whether edges tend to be “more prevalent” between high-degree nodes or not

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But! In most networks this won’t look particularly interesting – high-degree nodes are more likely to be connected to each other just by chance

So really, we need to compare the above distribution to that of a random model

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We need to compare the observed graph with a “random” graph that has the same degree distribution

- We’ve seen already how designing random graph models that preserve the correct statistics is **hard**
- But it turns out that generating a random sample with a **specific** degree distribution is **easy!**

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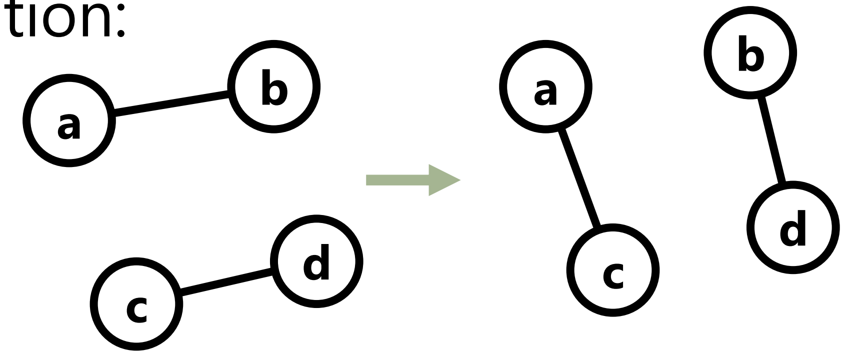
How to randomly sample graphs with a specific degree distribution:

- Start with any graph with the correct degree distribution (e.g. the input graph we're trying to assess)
- Randomly select pairs of (non-adjacent) edges (a→b) and (c→d)
- Apply the following operation:
- Repeat many times



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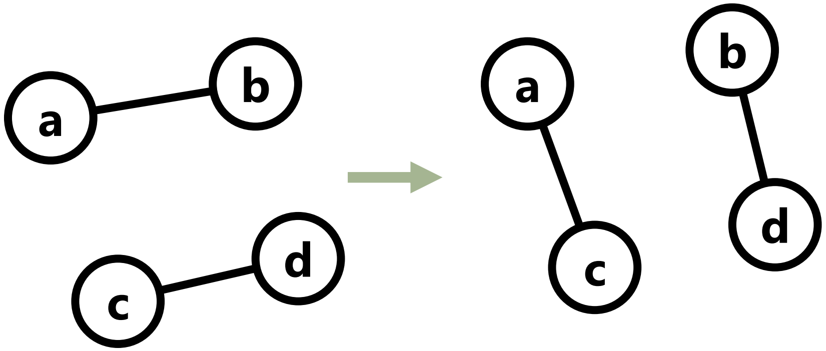
How to randomly sample graphs with a specific degree distribution:

- This operation **preserves the degree distribution**



- If the operation is repeated enough times, then **any** graph with the desired degree distribution is equally likely to be observed (i.e., we're sampling uniformly; see Milo et al.)

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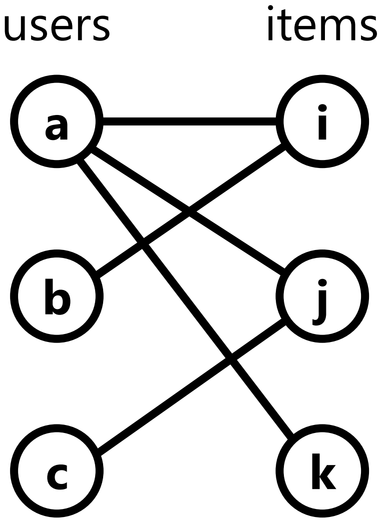
(aside: this is also useful when training recommender systems!)

e.g. when training a system to predict likely purchases for each user we wouldn’t want the system to be biased toward “power users”. To get around this, we may generate a “negative set” for training purposes that has the same degree distribution as the positive set. We can do this by applying the previous procedure to the bipartite purchasing graph



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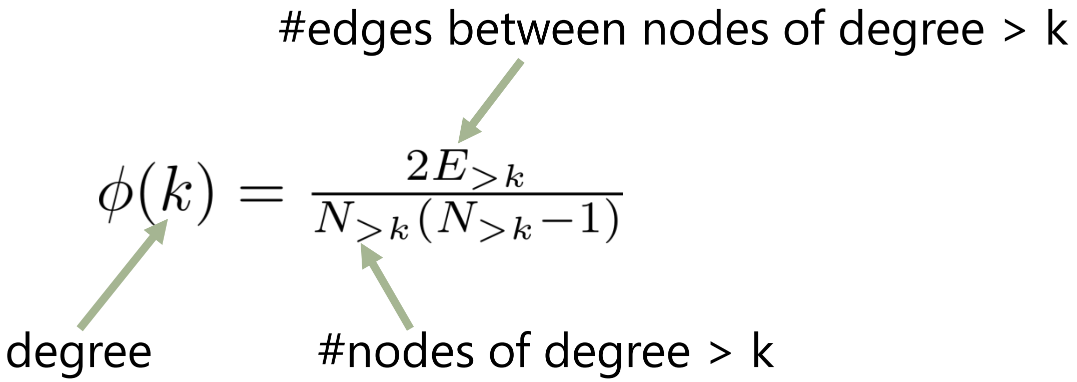
We can now compare the statistics of the randomly sampled graph to the statistics of the observed graph



#edges between nodes of degree > k

$$\phi(k) = \frac{2E_{>k}}{N_{>k}(N_{>k}-1)}$$

degree

#nodes of degree > k

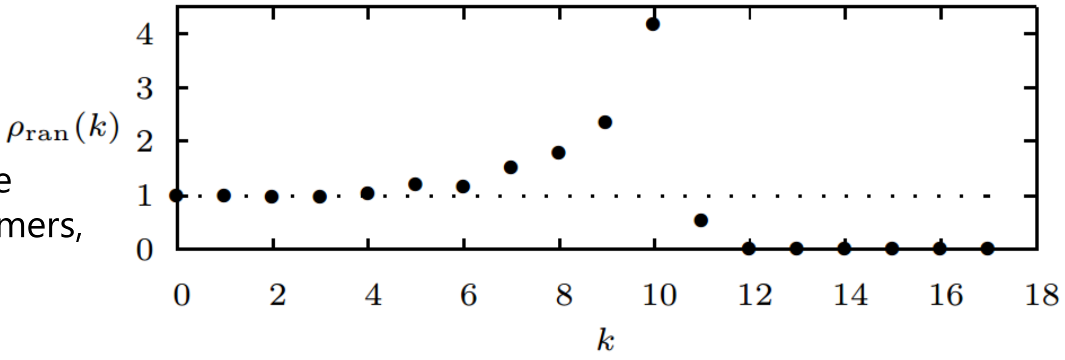
i.e., we compute

$$\phi_{\text{obs}}(k)/\phi_{\text{rand}}(k)$$

Some results

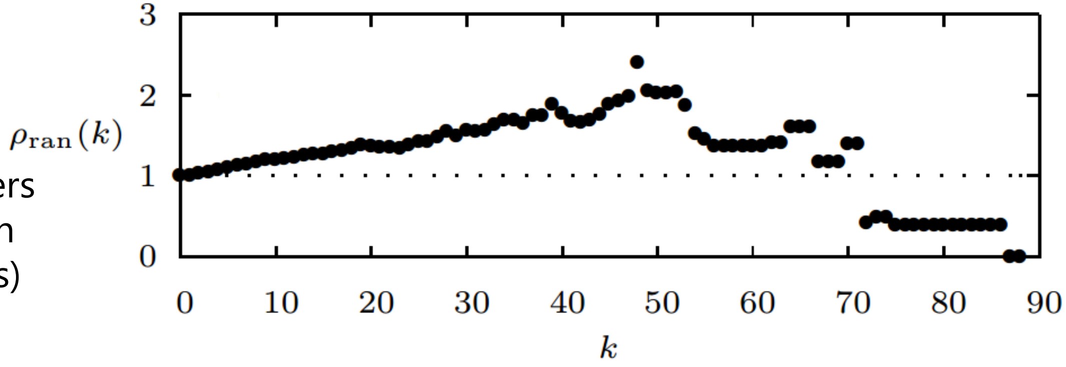
Power grid
(Watts & Strogatz, 1998)

(high-degree plants/transformers tend to be connected to other high-degree plants/transformers, except for the most connected plants)



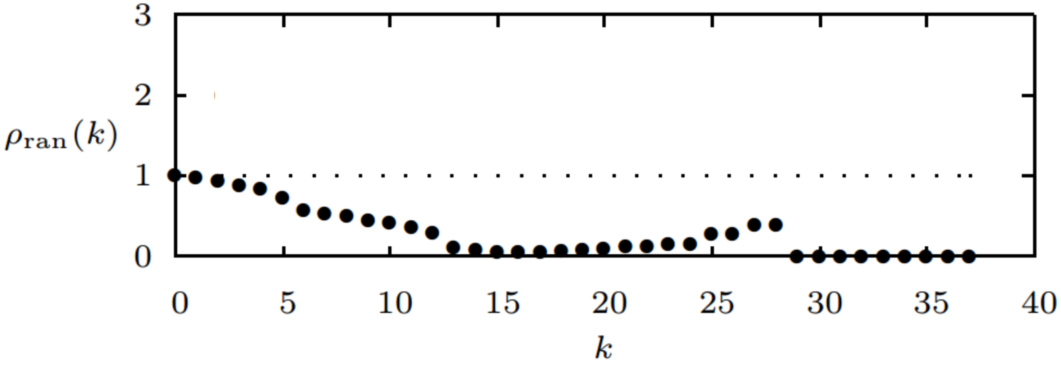
Scientific collaborations
(Colizza et al. 2006)

(highly-connected authors tend to write papers with other highly connected authors, though perhaps not for the **most** connected authors)



PPI network
(Jeong et al. 2001)

(highly-interactive proteins (e.g. regulators) tend **not** to interact with each other)



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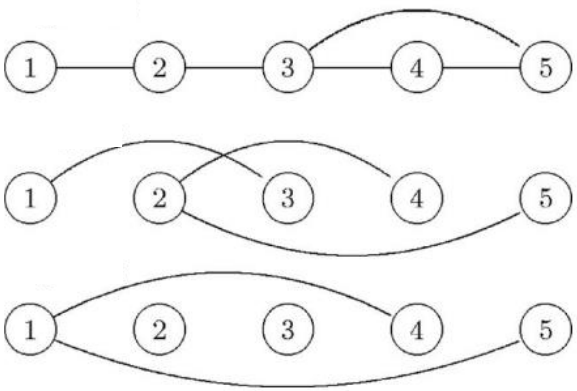
What about network **hierarchies?**

- So far we've looked at the likelihood that high-degree nodes are connected to other high-degree nodes
- But what about the probability that high-degree nodes have **paths** to other high degree nodes
- e.g. do high-degree nodes tend to be two-hops away from other high-degree nodes?

To test this we construct "higher-order" networks (by taking powers of the adjacency matrix)



original network

2nd-order network

3rd-order network

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Rich-clubs in higher-order networks

Why (part 2)?

- Perhaps in citation networks there are authors who act as “bridges” between the highest degree nodes, e.g. people who operate at the intersection between two areas
- Perhaps we can better characterize robustness to failure in a power grid, e.g. the distances between critical points of failure
- Or in a PPI networks, while we saw that “regulators” don’t tend to interact, perhaps some proteins act as “intermediaries” between regulators

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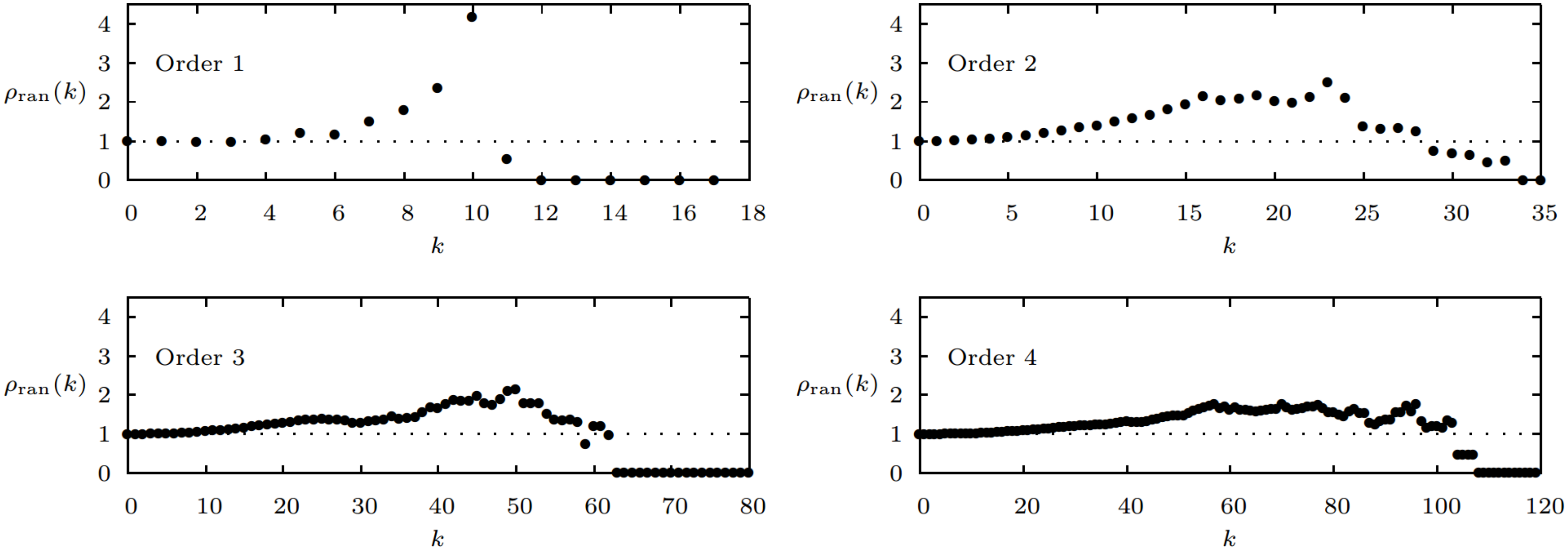
Rich-clubs in higher-order networks



Power grids: roughly the same sort of behavior exists across all orders of the network

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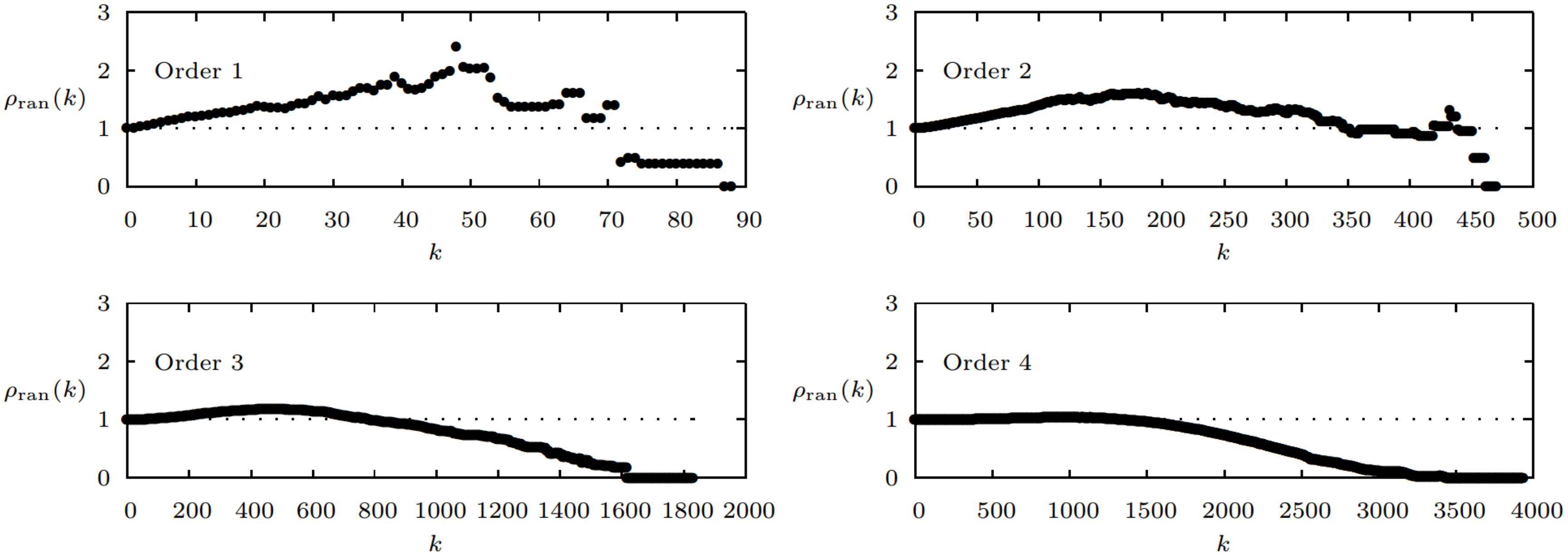
Rich-clubs in higher-order networks



Citation networks: rich-club behavior gradually disappears in higher-order networks

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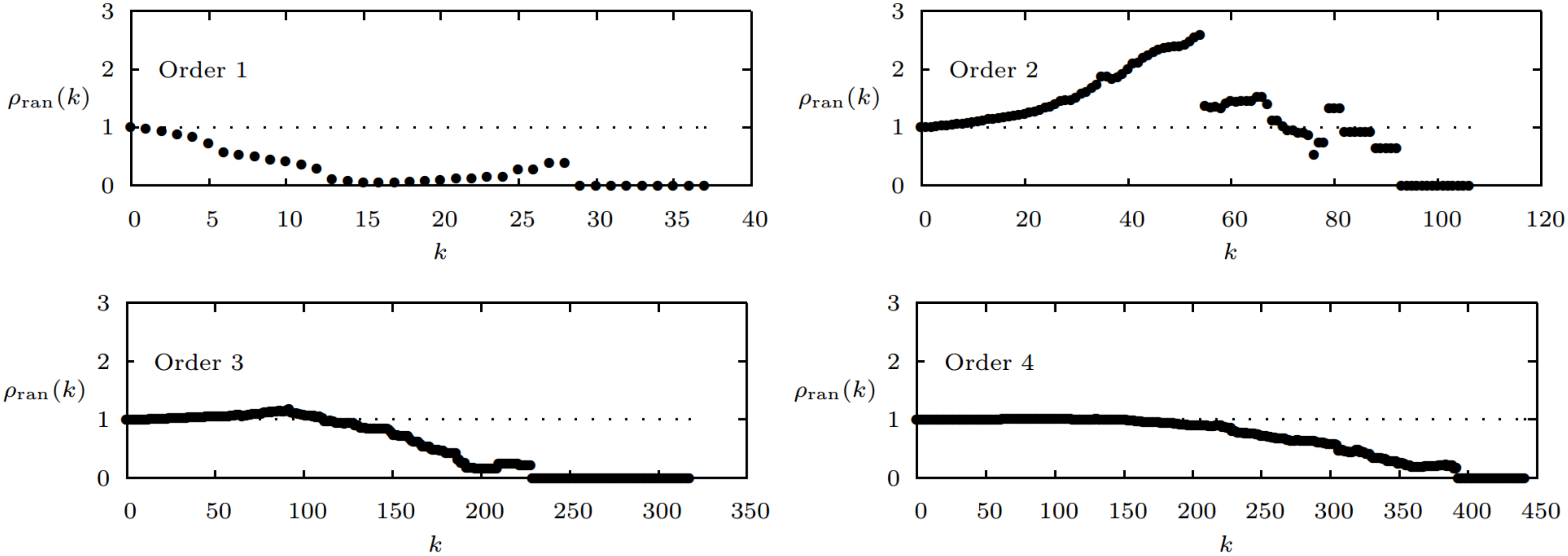
Rich-clubs in higher-order networks



PPI networks: the phenomenon behaves **non-monotonically:** there are few interacting hubs, but hubs seem to have intermediaries

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Morals of the story:

- The rich-club phenomenon can reveal important structural properties of a network, such as the roles of high-degree nodes and the networks resilience against cascading failures
- To understand whether high-degree nodes connect to each other, we need to compare against the right random model
- Rich-clubs in **higher-order** networks can further reveal the extent to which high-degree nodes may be connected via intermediaries

Questions?

The **final homework** is available on the course webpage:
http://cseweb.ucsd.edu/~jmcauley/cse255/homework9.pdf
If you want it back, swing by during office hours

Course evaluations are also online!